

# A MODEL OF SPARK AND LASER GENERATED BUBBLES

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A simple yet reasonably accurate model of spark and laser generated bubbles is presented and discussed. The behaviour of vapour bubbles is analysed and compared with that of the gas bubbles. The values of the significant bubble wall positions and of the related parameters computed for various initial pressures are also given in the paper.

## 1. INTRODUCTION

Vapour bubbles play a basic role in such important physical phenomena as cavitation and boiling [1–3]. In addition to this they are also generated during a number of other physical processes, such as electrical discharges in liquids (spark bubbles) [4–7], the irradiation of liquids by a focused laser light [8–12], the blowing of hot vapour into water [13], underwater explosions of detonating gases [14–16], and underwater nuclear explosions [17, 18].

While the cavitation bubbles are generated by a transient change in the ambient pressure [19], the remaining five processes mentioned above can be characterised by a common feature, namely that initially the bubble energy is violently increased [20]. Among the excitation methods based on increasing the vapour bubble energy, the spark and laser generated bubbles have received the greatest attention in the past, above all in connection with the laboratory modelling of cavitation bubbles [4–7, 9, 11, 12].

The object of this paper is to present a mathematical model of spark and laser generated bubbles that is reasonably accurate yet simple enough to allow fast computations. To make the exposition as simple as possible, only spherical, medium-sized bubbles will be considered, for which the effects of gravity, surface tension, viscosity, and heat conduction can be neglected [21].

## 2. BUBBLE MODEL

Let us assume that at a time  $t = 0$  a spherical portion of a liquid is converted into a vapour of a pressure  $P_{M0} \gg p_\infty$  and a temperature  $\Theta_{M0} \gg \Theta_\infty$ , where  $p_\infty$  and  $\Theta_\infty$  are liquid pressure and temperature for  $t < 0$ . Let the initial radius of the sphere be  $R_{m0}$ . The vapour bubble thus generated starts expanding in the same manner as a gas bubble [20, 22]. It will be further assumed that later on, when the bubble

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wall velocity decreases, the process of evaporation will maintain the pressure and temperature at the bubble wall equal to  $P = P_v$  and  $\Theta = \Theta_\infty$ , respectively, where  $P_v$  is the liquid vapour pressure.

After reaching the maximum radius,  $R_{M1}$ , the bubble wall motion reverts and the bubble enters a collapse phase [23]. At the beginning of this phase condensation will continue to maintain the equality  $P = P_v$  and  $\Theta = \Theta_\infty$ , but as the bubble wall velocity,  $\dot{R}$ , increases, condensation fails to keep pace with the wall motion and the bubble starts behaving as a gas bubble again [23]. The inward bubble wall motion is stopped by a violent pressure increase at a minimum radius,  $R_{m1}$ , and the direction of the bubble wall motion reverts. The new phase, which follows the collapse phase, is usually called the rebound phase. The collapse and rebound phases will successively recur several times during the bubble life.

To describe the bubble wall motion, Herring's modified equation [24] will be used. This equation has a form

$$(1) \quad \ddot{R}R + \frac{3}{2}\dot{R}^2 = \frac{1}{\varrho_\infty} [P - p_\infty + \dot{P}R/c_\infty].$$

Here  $\varrho_\infty$  is the liquid density,  $P$  is the pressure in the liquid at the bubble wall, and  $c_\infty$  is the velocity of sound in the liquid. The dots denote differentiation with respect to time.

For moderate bubble oscillations the pressure wave,  $p$ , at a point in the liquid,  $r$ , radiated by the bubble, can be determined from a simple relation [25] ( $r \gg R_{m0}$ )

$$(2) \quad p_a = p - p_\infty = (P - p_\infty + \frac{1}{2}\varrho_\infty\dot{R}^2) R/r.$$

Here  $p_a$  denotes the acoustic pressure. From equation (2) the peak pressure  $p_{pk}$  in the  $k$ -th bubble pulse is

$$(3) \quad p_{pk} = p_{Mk} - p_\infty = (P_{Mk} - p_\infty) R_{mk}/r, \quad k = 1, 2, \dots$$

Here  $P_{Mk}$  is the maximum pressure at the bubble wall in the  $k$ -th minimum bubble volume, i.e., when  $R = R_{mk}$ , and  $p_{Mk}$  is a maximum pressure in the  $k$ -th bubble pulse measured at the point  $r$  [25].

Let us note that in contrast to eq. (2), eq. (3) is valid even for violent bubble oscillations, but only as long as no shock develops in the wave [25].

To describe the behaviour of the vapour in the bubble a simple model given in reference [23] will be used. This model is based on the idea of a switching between an ideal vapour bubble and an ideal gas bubble, and it is assumed that in the ideal vapour bubble condensation and evaporation take place at an infinite speed and in the ideal gas bubble at zero speed. Evidently, the real bubble approaches the behaviour of the ideal vapour bubble for wall velocities,  $\dot{R}$ , which are lower than a certain limiting velocity,  $\dot{R}_{vg}$ , and it approaches the behaviour of the ideal gas bubble for wall velocities higher than  $\dot{R}_{vg}$ . Thus for the initial growth phase we have:

if  $\dot{R} > \dot{R}_{vg}$ , then

$$(4) \quad P = P_{M0}(R/R_{M0})^{-3\gamma},$$

and if  $\dot{R} \leq \dot{R}_{vg}$ , then

$$(5) \quad P = P_v.$$

In eq. (4),  $\gamma$  is the ratio of the specific heats for the vapour.

Similarly for the collapse and rebound phases we have: if  $|\dot{R}| \leq |\dot{R}_{vg}|$ , then eq. (5) is valid, and if  $|\dot{R}| > |\dot{R}_{vg}|$ , then

$$(6) \quad P = P_v(R/R_{vg})^{-3\gamma}.$$

As the equality  $\dot{R} = \dot{R}_{vg}$  occurs not only in the vicinity of  $R_M$ , but also of  $R_m$ , it is necessary to discriminate between the two areas in a computer program and to switch only in the vicinity of  $R_M$ . A second condition, such as  $P < p_\infty$ , can be used for this purpose. In the following only the switching position near  $R_M$  is considered.

For further work it is convenient to introduce non-dimensional expansion  $W$  variables. The non-dimensional time, radius, pressure in the liquid at the bubble wall, and pressure in the liquid at a point  $r$  are defined as, respectively [20, 22]

$$t_w = t/[R_{m0}(\rho_\infty/p_\infty)^{1/2}], \quad W = R/R_{m0}, \quad P^* = P/p_\infty, \\ p_w = p_a r / (p_\infty R_{m0}).$$

Using these variables eqs. (1) to (6) can be normalized to take the form

$$(7) \quad \dot{W}W + \frac{3}{2}W^2 = P^* - 1 + \dot{P}^*W/c_\infty^*,$$

$$(8) \quad p_w = (P^* - 1 + \frac{1}{2}W^2)W,$$

and

$$(9) \quad p_{wpk} = (P_{Mk}^* - 1)W_{mk}, \quad k = 1, 2, \dots$$

During the growth phase, if  $\dot{W} > \dot{W}_{vg}$ , then

$$(10) \quad P^* = P_{M0}^* W^{-3\gamma},$$

and if  $\dot{W} \leq \dot{W}_{vg}$ , then

$$(11) \quad P^* = P_v^*.$$

During the collapse and rebound phases, if  $|\dot{W}| > |\dot{W}_{vg}|$ , then

$$(12) \quad P^* = P_v^*(W_{vg}/W)^{3\gamma},$$

and if  $|\dot{W}| \leq |\dot{W}_{vg}|$ , eq. (11) holds.

In eq. (7) the non-dimensional sound velocity is defined as  $c_\infty^* = c_\infty(\rho_\infty/p_\infty)^{1/2}$  and the initial conditions are  $W(0) = 1$ ,  $\dot{W}(0) = 0$ .

The computations will be performed for the following values of physical constants (water under ordinary laboratory conditions):

$$P_{\infty} = 100 \text{ kPa}, \quad \rho_{\infty} = 10^3 \text{ kg m}^{-3}, \quad c_{\infty} = 1450 \text{ m s}^{-1},$$

$$P_v = 2 \text{ kPa}, \quad |\dot{R}_{vg}| = 6 \text{ m s}^{-1}, \quad \gamma = 1.25.$$

An example of the computed bubble wall motion time history is displayed in fig. 1, and of the radiated pressure wave in fig. 2. These graphs were computed using eqs. (7), (8) and (10)–(12). Let us note that for intensities of the bubble oscillations considered here eq. (8) does not give correct results for all pressures. It yields correct values for  $P^* = P_{Mk}^*$  and for  $P^* \leq 1$  [25]; for pressures  $P_{Mk}^* > P^* > 1$  there will

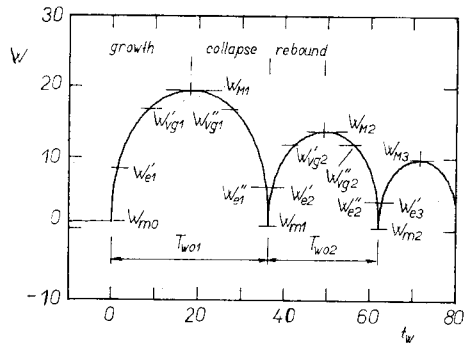


Fig. 1. Time history of the vapour bubble wall motion.  $P_{M0}^* = 3000$ .  $T_{w0}$  — time of bubble oscillation.

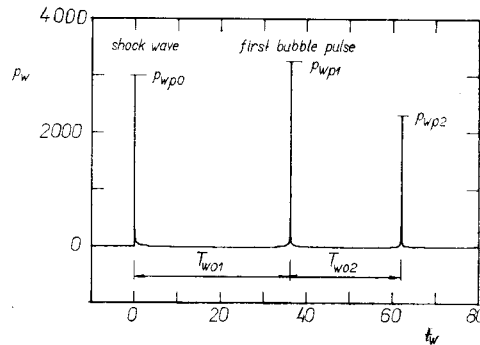


Fig. 2. Pressure wave radiated by an oscillating vapour bubble.  $P_{M0}^* = 3000$ .  $T_{w0}$  — time of bubble oscillation.

be a difference between the “actual” and the computed waveform. However, with regard to the time and pressure resolution used in displaying the waveform in fig. 2 the difference will hardly be noticeable.

Finally let us note that a vapour bubble model similar to the one given here was also used by Kimoto et al. [12]. These authors assumed a switching velocity  $|\dot{R}_{vg}| = 10 \text{ m s}^{-1}$ , liquid vapour pressures  $P_v = 30, 51, \text{ and } 71 \text{ kPa}$ , and started their computations at  $R_{M1}$  (i.e., they did not consider the growth phase).

### 3. ANALYSIS OF THE BUBBLE BEHAVIOUR

To gain a better understanding of the bubbles' behaviour computations have been carried out for three initial pressures  $P_{M0}^* = 3000, 10000, \text{ and } 30000$ . The calculations were terminated at the third maximum bubble radius  $W_{M3}$ . Values of the significant bubble wall positions thus found are summarized in table 1. The values of the physical constants used were given in section 2. For definition of the significant positions see fig. 1 and the previous section. The equilibrium radius,  $W_e$ , was determined from the condition that  $P^* = 1$  when  $W = W_e$  [25].

From the values of the computed significant positions a number of related quantities were determined. These are summarized in table 2.

Table 1  
Computed significant bubble wall positions.

$P_{M0}^*$	3 000	10 000	30 000
$W_{m0}$	1.00	1.00	1.00
$W'_{e1}$	8.46	11.66	15.63
$W'_{vg1}$	16.81	23.59	31.69
$W_{M1}$	19.46	27.31	36.68
$W''_{vg1}$	16.81	23.59	31.69
$W''_{e1}$	5.92	8.31	11.17
$W_{m1}$	0.60	0.84	1.13
$W'_{e2}$	5.92	8.31	11.17
$W'_{vg2}$	11.92	16.73	22.46
$W_{M2}$	13.80	19.36	26.00
$W''_{vg2}$	11.92	16.73	22.46
$W''_{e2}$	4.20	5.89	7.91
$W_{m2}$	0.42	0.59	0.80
$W'_{e3}$	4.20	5.89	7.91
$W_{M3}$	9.78	13.72	18.43

Table 2.  
Computed significant bubble parameters.

$P_{M0}^*$	3 000	10 000	30 000
$A'_1$	2.30	2.34	2.35
$A''_1$	3.29	3.29	3.29
$P_{M1}^*$	5 460	5 460	5 460
$P_{wp1}$	3 260	4 575	6 140
$P_{zp1}$	167	167	167
$A'_2$	2.33	2.33	2.33
$\alpha_1$	0.71	0.71	0.71
$A''_2$	3.29	3.29	3.29
$P_{M2}^*$	5 460	5 460	5 460
$P_{wp2}$	2 310	3 240	4 355
$P_{zp2}$	119	119	119
$A'_3$	2.33	2.33	2.33
$\alpha_2$	0.71	0.71	0.71

In table 2 the growth amplitudes,  $A'_1 = W_{M1}/W'_{e1}$ , and rebound amplitudes,  $A'_2 = W_{M2}/W'_{e2}$ ,  $A'_3 = W_{M3}/W'_{e3}$ , represent the amplitudes for which the bubbles would be excited, if they were gas bubbles. These amplitudes are determined by the maximum pressures  $P_{M0}^*$ ,  $P_{M1}^*$ , and  $P_{M2}^*$ , respectively [22].

However, thanks to the collapse mechanism, the amplitudes  $A''_1 = W_{M1}/W''_{e1}$ , and  $A''_2 = W_{M2}/W''_{e2}$ , which govern the collapse phase, are larger than the growth and rebound amplitudes, i.e.  $A''_1 > A'_1$ , and  $A''_2 > A'_2$ . Note also that the collapse amplitudes equal each other and are independent of the maximum pressures  $P_{M0}^*$ , and  $P_{M1}^*$ . Similarly the damping factors,  $\alpha_1 = W_{M2}/W_{M1}$ , and  $\alpha_2 = W_{M3}/W_{M2}$ , equal each other.

The parameter  $R_{m0}$  represents a computational quantity only, with no simple physical meaning. It cannot be determined directly in experiments, and therefore for evaluation of the experimental data, it is necessary to use the  $Z$  system based on the maximum radius  $R_{M1}$  [26]. When transforming the values of the significant wall positions and peak pressures in table 1 and 2 from the  $W$  into the  $Z$  system, simple formulae can be used, namely  $Z = W/W_{M1}$  and  $p_{zp} = p_{wp}/W_{M1}$ .

In table 2 only the peak pressures are given in both systems. This is so because of the importance of the pressure  $p_{zp}$ , which is more suitable than  $p_{wp}$  for comparison of the bubbles excited by different initial pressures  $P_{M0}^*$  and for comparison of theoretical values with experimental data [26].

It is also interesting to compare the vapour and gas bubbles. Thus for the **gas bubbles** it always holds that [27] (see also figs. 1 and 2 in reference [22])

$$(13) \quad W_{m0} < W_{m1} < W_{m2} < \dots,$$

$$(14) \quad p_{wp0} > p_{wp1} > p_{wp2} > \dots$$

However, in the case of the **vapour bubbles** the situation is more complex, and, according to the value of the initial pressure  $P_{M0}^*$ , three situations may occur (cf. tables 1 and 2). For relatively low pressures  $P_{M0}^*$  it was determined that

$$(15) \quad W_{m0} > W_{m1} > W_{m2} > \dots,$$

$$(16) \quad p_{wp0} < p_{wp1} > p_{wp2} > \dots$$

In the case of medium high pressures  $P_{M0}^*$  one has

$$(17) \quad W_{m0} > W_{m1} > W_{m2} > \dots,$$

$$(18) \quad p_{wp0} > p_{wp1} > p_{wp2} > \dots,$$

and finally for high pressures  $P_{M0}^*$  it holds that

$$(19) \quad W_{m0} < W_{m1} > W_{m2} > \dots,$$

$$(20) \quad p_{wp0} > p_{wp1} > p_{wp2} > \dots$$

The inequalities (15)–(20) are due to the amplifying collapse effect, and the relations  $p_{wp0} < p_{wp1}$  and  $W_{m0} > W_{m1}$ , which can exist only in the case of the vapour

bubbles, may be used to discriminate between the gas and vapour bubbles in experiments.

A further difference between the pure vapour and gas bubbles exists in the behaviour of the equilibrium radii and amplitudes of the successive oscillations. Whereas for the **gas bubbles** it holds that

$$(21) \quad W_{e1} = W_{e2} = W_{e3} = \dots,$$

$$(22) \quad A_1 > A_2 > A_3 > \dots,$$

for the pure **vapour bubbles** we can write that

$$(23) \quad W'_{e1} > W''_{e1} = W'_{e2} > W''_{e2} = W'_{e3} > \dots,$$

$$(24) \quad A'_1 = A''_2 = A''_3 = \dots,$$

$$(25) \quad A'_2 = A'_3 = \dots$$

However, let us note that these relations hold only for the vapour bubble models that do not take into account viscosity, surface tension, and heat conduction. In a real environment, even if the vapour bubble begins oscillating with the size of a scaling bubble, for which the effects mentioned are not important [21], due to the inequality  $W'_{e1} > W'_{e2} > \dots$ , the bubble will successively shrink to a size where these mechanisms become important and hence the equalities  $A'_2 = A'_3 = \dots$ ,  $A'_1 = A'_2 = \dots$  cease to be valid.

Equally important is the fact that due to the rectified diffusion of the gases from the liquid into the bubble interior [28] the vapour bubble will gradually behave more and more like a gas bubble, which means that relations (21) and (22) will prevail above relations (23)–(25). As shown in [29], experimental data published in the literature show that the spark and laser generated bubbles usually behave like pure vapour bubbles only for a very short time interval (say, during the first oscillation) and then gradually assume a behaviour similar to that of gas bubbles.

To stress the difference between the gas and vapour bubbles, a different terminology is used for the bubble life phases in the two cases. Whereas for the gas bubbles terms such as expansion and compression phases have been adopted [25], in the case of the vapour bubbles terms such as growth, collapse, and rebound phases are preferred.

#### 4. CONCLUSION

Although the bubble model introduced is rather simple, it will be shown elsewhere [29] that in many respects it describes the behaviour of real bubbles rather well. Evidently, the model does not deal with the initial instants of the spark and laser generated bubbles adequately. Similarly its validity is weakened in the vicinity of the first bubble volume minimum and later on. The use of the switching condition,  $\dot{R} = \dot{R}_{vg}$ , for the growth and rebound phases is also questionable. However, a great

advantage of the model lies in its computational simplicity and economy, which outweighs to some degree these inaccuracies.

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