The Scaling Law for Free Oscillations of Gas Bubbles *

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Summary

The principle of similarity and the scaling law are formulated for free oscillations of gas bubbles and the conditions under which the scaling law can be applied are examined. The limits for the effects of gravity, surface tension, and viscosity are computed. The limits for the effect of heat losses are estimated from published data on the linear bubble oscillations and from experimental data published by Mellen [18].

Das Skalierungsgesetz für freie Schwingungen von Gasblasen

Zusammenfassung

Es werden das Ähnlichkeitsprinzip und das Skalierungsgesetz für freie Schwingungen von Gasblasen formuliert und die Bedingungen untersucht, unter welchen das Skalierungsgesetz angewandt werden kann. Die Grenzen für die Auswirkungen der Schwere, der Oberflächenspannung und der Viskosität werden berechnet. Weiterhin werden die Grenzen für Wärmeverluste aufgrund von publizierten Werten über lineare Blasenschwingungen sowie von experimentellen Daten nach Mellen [18] abgeschätzt.

Une loi de calibrage pour les oscillations libres des bulles de gaz

Sommaire

On formule le principe de similitude et la loi de calibrage pour des oscillations libres de bulles de gaz, en précisant les conditions d'application. On calcule des valeurs limites pour les effets de la gravité, de la tension superficielle et de la viscosité. On évalue également les limitations concernant l'effet des pertes thermiques, en s'appuyant sur des données expérimentales publiées par Mellen et sur les données théoriques admises pour les oscillations linéaires des bulles.

1. Introduction

The principle of similarity has been successfully used in many branches of science and technology for a long time (see, e.g. [1−3]). Although it has also been used in cavitation research in connection with the modelling of hydraulic equipment [4], it has been only very rarely mentioned in theoretical literature or used in experimental works on bubble dynamics. The few exceptions known to the author are works [5−9]. However, it is our conviction that a special form of this principle, namely the scaling law, can be conveniently used in the study of freely oscillating bubbles.

* This work is taken from the author's Ph.D. dissertation [20]. In shortened form it was presented at the seminar on cavitation [21].

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The object of this paper is to formulate the principle of similarity and the scaling law for free oscillations of gas bubbles and to determine the conditions under which these laws can be applied.

In our analysis a spherical gas bubble is assumed to oscillate radially in an infinite volume of a liquid. As we are primarily interested in laboratory experiments conducted under ordinary conditions, the numerical computations will be carried out for water and for the following values of the physical constants:

Pressure in the liquid at infinity	p_∞	$= 100\,\mathrm{kPa}$,
Density of water	ϱ_∞	$= 10^3\,\mathrm{kg\,m^{-3}}$,
Velocity of sound in water	c_∞	$= 1450\,\mathrm{m\,s^{-1}}$,
Acceleration of gravity	g	$= 10\,\mathrm{m\,s^{-2}}$,
Surface tension of water	σ	$= 75 \cdot 10^{-3}\,\mathrm{N\,m^{-1}}$,
Shear viscosity of water	η	$= 10^{-3}\,\mathrm{kg\,s^{-1}\,m^{-1}}$,

The constants in the Tait equation of state for water $B = 300$ MPa, $n = 7$.

2. The principle of similarity and the scaling law

A mathematical model of an oscillating bubble is represented by a system of equations that make it possible to determine the bubble wall motion, the pressure, velocity, and temperature fields, and other quantities of interest inside and outside the bubble. Solution of such a model has the form

$$\begin{aligned} R &= R(t; R_0, P_0; \gamma, p_\infty, \varrho_\infty, \ldots), \\ p &= p(t, r; R_0, P_0; \gamma, p_\infty, \varrho_\infty, \ldots), \\ &\vdots \end{aligned} \qquad (1)$$

Here R is the bubble wall radius at a time t, p is the pressure in the liquid at a time t and at a point r. R_0 and P_0 are the characteristic bubble radius and the gas pressure at the bubble wall, respectively, and finally, γ is the ratio of the principal specific heats.

As denoted in relations (1), the dependent variables $R, p, \ldots$ are functions of independent variables t, r, and of several parameters. These parameters may be divided into two groups. The first group encompasses the parameters that describe the bubble itself, i.e. its size (R_0) and intensity of its oscillations (P_0). The second group relates to the physical properties of the liquid, gas in the bubble interior, and so on.

As discussed elsewhere [10], the bubble size and intensity of oscillations can be described in several ways, the most suitable one being dependent on a concrete situation under consideration. Here the maximum bubble radius, R_M, will be used as a measure of the bubble size, and a non-linear amplitude, $A = R_M/R_e$, as a measure of the bubble oscillation intensity (R_e is an equilibrium radius).

Two models of the freely oscillating gas bubble will be considered in this study. These are: (i) Herring's modified model [11], and (ii) Gilmore's model [11]. In these models the bubble environment is characterized by the physical constants $\gamma, p_\infty, \varrho_\infty, c_\infty, g, \sigma, \eta, B$, and n, already defined in Section 1.

To obtain results in a general form suitable for the formulation of the similarity principle the basic equations representing the bubble models will be normalized by introducing suitable non-dimensional variables. Here we shall primarily use the compression variables [10, 12]

$$t_z = \frac{t}{R_M \sqrt{\dfrac{\varrho_\infty}{p_\infty}}}, \quad Z = \frac{R}{R_M}, \quad z = \frac{r}{R_M},$$

$$P^* = \frac{P}{p_\infty}, \quad p^* = \frac{p}{p_\infty},$$

and in Section 3 in part the expansion variables [10, 12]

$$t_w = \frac{t}{R_m \sqrt{\dfrac{\varrho_\infty}{p_\infty}}}, \quad W = \frac{R}{R_m}.$$

Here R_m is the minimum bubble radius.

If in the following we limit ourselves only to the wall-motion and to the pressure field, in the compression system the solution (1) can be written in the form

$$Z = Z(t_z; A; \gamma, c_\infty^*, g_z, \sigma_z, \eta_z, B^*, n), \qquad (2)$$

$$p^* = p^*(t_z, z; A; \gamma, c_\infty^*, g_z, \sigma_z, \eta_z, B^*, n), \qquad (3)$$

where

$$c_\infty^* = c_\infty \sqrt{\frac{\varrho_\infty}{p_\infty}}, \quad g_z = g R_M \frac{\varrho_\infty}{p_\infty}, \quad \sigma_z = \frac{\sigma}{R_M p_\infty},$$

$$\eta_z = \frac{\eta}{R_M \sqrt{p_\infty \varrho_\infty}}, \quad B^* = \frac{B}{p_\infty}.$$

Note that the non-dimensional quantities whose pi groups contain the bubble size R_M are denoted by a letter or subscript z, whereas the other non-dimensional quantities are denoted by an asterisk [12].

Now the *principle of similarity* asserts (freely adapted from [1]) that *two oscillating bubbles which are so related to each other that the arguments inside the functional signs in (2) and (3) are equal numerically, form two physically similar systems.*

It is our opinion that in this general form the principle of similarity has limited use in the bubble dynamics studies. However, as the magnitude of the non-dimensional parameters g_z, σ_z, and η_z, depends on R_M, it may be expected that for certain bubble sizes these parameters will have negligible influence. If this is the case they can be omitted from eqs. (2) and (3) and we obtain

$$Z = Z(t_z; A; \gamma, c_\infty^*, B^*, n), \qquad (4)$$

$$p^* = p^*(t_z, z; A; \gamma, c_\infty^*, B^*, n). \qquad (5)$$

If the pressure changes in the liquid are moderate (acoustic waves) and the ambient pressure p_∞ is much less than B the liquid can be taken as linear and the parameters B^* and n can also be omitted. Finally, if we limit ourselves to the acoustic field ($z \gg 1$), or to the peak and valley values in the pressure wave [12], eqs. (4) and (5) can be further simplified. Defining the acoustic position independent pressure as $p_z = (p^* - 1)\, z$ [12], we obtain

$$Z = Z(t_z; A; \gamma, c_\infty^*), \qquad (6)$$

$$p_z = p_z(t_z; A; \gamma, c_\infty^*). \qquad (7)$$

If we further assume the same physical conditions for both bubbles (i.e. the same values of $\gamma, p_\infty, \varrho_\infty$,

c_∞), then the only parameters that can be varied in (6) and (7) are the amplitude, A, and the bubble size, R_M, and we can reformulate the principle of similarity for the free bubble oscillations in the following special form (freely adapted from [5]):

The wall motions and the acoustic fields of two bubbles oscillating with the same amplitude A will be the same if the scales of length and time by which these quantities are measured are changed by the same factor, λ, by which the characteristic dimensions, R_0, of these two bubbles differ.

This special form of the similarity principle, where the only variable quantity is the bubble size, will be referred to as the *scaling law*.

In a broader sense the scaling law can be formulated even for the kinetic field [12] and the finite-amplitude waves (eq. (5)). However, in this case it is necessary for the argument z to have the same value for the two bubbles as well.

Finally let us note that the scaling law could also be formulated for further quantities. Under above given conditions some of these quantities do not even change their values with the change of the bubble size (the so-called independent quantities [12]).

The bubbles satisfying the scaling law will be called the *scaling bubbles*.

3. Limits

In deriving eqs. (4) to (7) the basic assumption involved omission of the parameters g_z, σ_z, and η_z from the equation of motion. It is the purpose of this section to find out the bubble sizes and amplitudes for which this omission can be justified.

For Rayleigh's model the respective limiting bubble sizes have been found in [12]. However, because in this model the liquid is assumed to be incompressible, the computations were carried out only for small amplitudes ($A \leqq 2$), for which Rayleigh's model gives satisfactory results [11]. In this section we shall present results of computations performed with Herring's modified and Gilmore's models, which are suitable even for large amplitudes [11].

As gravity is important only for macrobubbles and the surface tension and viscosity only for microbubbles, these effects will be examined separately.

3.1. Influence of gravity

In determining the limiting bubble size, R'_M, which represents a boundary on the bubble map for the effect of gravity, Herring's modified model [11] will be used. The modification consists in the omission of the correction terms from the equation of motion of Herring's original model. Comparison with the more exact Gilmore model shows that Herring's modified model gives satisfactory results for amplitudes approximately up to $A \leqq 4.5$ [11].

The effect of gravity will be respected through Taylor's term [5, 8, 12]. However, as we are primarily interested in laboratory experiments, the ambient pressure, p_∞, will be assumed to be constant here [12].

The equation of motion for Herring's modified model supplemented with Taylor's gravity term has the form

$$\ddot{R}R + \frac{3}{2}\dot{R}^2 = \frac{1}{\varrho_\infty}\left[P - p_\infty + \frac{R}{c_\infty}\dot{P}\right] + \frac{U^2}{4}, \qquad (8)$$

where the gas pressure at the bubble wall, P, equals

$$P = P_0\left(\frac{R}{R_0}\right)^{-3\gamma}. \qquad (9)$$

Here R_0 and P_0 are the characteristic (in this case the initial) bubble radius and gas pressure, respectively. The upward velocity, U, equals

$$U = \frac{2g}{R^3}\int_0^t R^3\,\mathrm{d}t. \qquad (10)$$

Though computations should be performed for each excitation technique separately, we have performed them only for the excitation by decreasing bubble energy, and excitation by increasing bubble energy [10]. In the case of excitation by decreasing bubble energy it is convenient to work with the compression (Z) system of non-dimensional variables, whereas in the case of excitation by increasing bubble energy the expansion (W) system of non-dimensional variables is appropriate [10, 12].

Eq. (8) was solved numerically in both systems for a number of values R_0 and P_0. The computations were carried out up to the second maximum radius. A number of computed significant values was recorded. In the compression system these were Z_{m1}, T_{zc1}, Z_{M2}, and T_{ze2} (Fig. 1a). In the expansion system we have recorded W_{M1}, T_{we1}, W_{m1}, T_{wc1}, W_{M2}, and T_{we2} (Fig. 1b). From these values the peak pressures in the waves, p_{zp1} and p_{wp1}, were calculated according to the formulae [12]

$$p_{zp1} = (P^*_m Z_{m1}^{-3\gamma} - 1)\, Z_{m1}, \qquad (11)$$

$$p_{wp1} = P^*_M W_{m1}^{-3\gamma} - 1. \qquad (12)$$

Here the pressures P^*_m and P^*_M are the initial pressures (P^*_0) in the compression and expansion systems, respectively.

Finally, the damping factor, α_1, was determined from the relation

$$\alpha_1 = Z_{M2} = \frac{W_{M2}}{W_{M1}}. \tag{13}$$

It was found that times T_{zc1}, T_{ze1}, T_{we1}, T_{wc1}, and T_{we2} vary only little with R_0. The same is true for α_1. Therefore, these quantities will not be considered further. This enables us to simplify the notation by dropping the digits from the subscripts.

From the computed values of Z_m, p_{zp}, W_m, and p_{wp}, a relative deviation, δ, was calculated using the formula

$$\delta = \frac{M-N}{N} 100\,[\%]. \tag{14}$$

Here N represents the values computed when the effect of gravity was negligible (small bubble sizes R_M and R_m), and M stands for values corresponding to larger bubble sizes R_M and R_m.

Calculated variations of δ_z, δ_w, and δ_p, with R_M, are depicted in Fig. 2. From these graphs it is possible to determine the limiting bubble sizes, R'_M, corresponding to the given value of δ. The results thus obtained for $\delta = 5\%$ are displayed in Fig. 3. To enable us to make an easy comparison, the results are displayed in the co-ordinate system "amplitude – size" (the bubble map). Here we have used the formulae $P_m^* = A^{-3\gamma}$, $P_M^* = (W_M/A)^{3\gamma}$, and $R_M = W_M R_m$ [10, 12].

In [12] the following approximate formula for R'_M was found

$$R'_M = K Z_m^{1.5}, \quad A \geqq 2, \tag{15}$$

where the constant K in the compression system equals $K = 2$. It is easy to show that the same formula holds even in the expansion system. However, now $K = 1$. The limits calculated according to formula (15) are also displayed in Fig. 3.

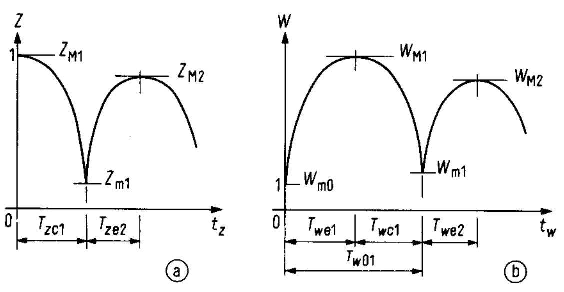

Fig. 1. Significant positions of the bubble-wall: a) Compression system, b) Expansion system.

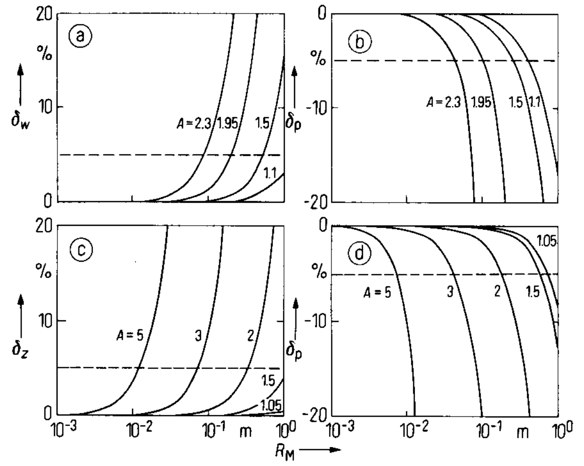

Fig. 2. Influence of gravity: (a) variation of the minimum bubble radius, W_{m1}, (b) variation of the peak pressure, p_{wp}, (c) variation of the minimum bubble radius, Z_{m1}, (d) variation of the peak pressure, p_{zp}, with the bubble size, R_M, and the amplitude, A. (a), (b) – expansion system, (c), (d) – compression system, $\gamma = 4/3$.

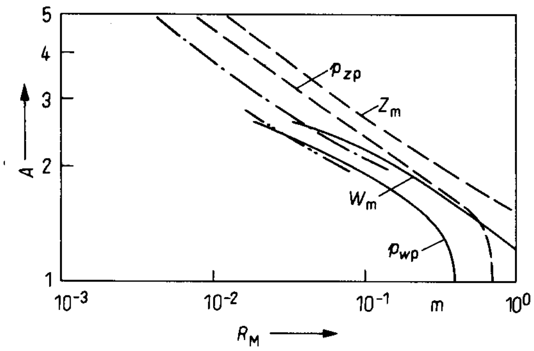

Fig. 3. Boundaries for the effect of gravity on the minimum radii, Z_m and W_m, and peak pressures, p_{zp} and p_{wp} ($\gamma = 4/3$): --- compression system, —— expansion system. Computation according to the approximate formula (15) ($\gamma = 4/3$): –·– compression system, –··– expansion system.

3.2. *Influence of the surface tension and viscosity*

In determining the limiting bubble size, R''_M, which represents the boundaries in the bubble map for the effects of the surface tension and viscosity, Gilmore's model will be used. The equation of motion in this model has the form [11]

$$\ddot{R}R\left(1-\frac{\dot{R}}{C}\right)+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3C}\right) = H\left(1+\frac{\dot{R}}{C}\right)+\frac{R}{C}\dot{H}\left(1-\frac{\dot{R}}{C}\right), \tag{16}$$

where the sound velocity in the liquid at the bubble wall, C, is given by

$$C = c_\infty \left(\frac{P+B}{p_\infty+B}\right)^{\frac{n-1}{2n}}, \tag{17}$$

and the enthalpy difference, H, equals

$$H = \frac{1}{\varrho_\infty} \frac{n}{n-1} (p_\infty + B) \left[\left(\frac{P+B}{p_\infty + B} \right)^{\frac{n-1}{n}} - 1 \right]. \quad (18)$$

The gas pressure at the bubble wall, P, in the presence of the surface tension and viscosity equals [12]

$$P = P_0 \left(\frac{R}{R_0} \right)^{-3\gamma} - \frac{2\sigma}{R} - 4\eta \frac{\dot{R}}{R}. \quad (19)$$

Though computations should be performed for each excitation technique separately, we have carried them out only in the compression system. Hence $P_0 = P_m$ and $R_0 = R_M$. Because of the surface tension the relation between A and P_m^* now has the form [12]

$$P_m^* = (1 + 2\sigma_z A) A^{-3\gamma}. \quad (20)$$

Eqs. (16) to (19) were normalized using the Z non-dimensional variables and solved for a number of amplitudes, A, bubble sizes, R_M, and for values of $\gamma = 4/3$ (adiabatic behaviour of the bubble), and $\gamma = 1$ (isothermal behaviour). The computations were carried out up to the second maximum radius Z_{M2} (Fig. 1a), and the same significant values as in the case of gravity were recorded (i.e. Z_{m1}, T_{zc1}, Z_{M2}). The peak pressure in the wave, p_{zp1}, was computed from the expression [12]

$$p_{zp1} = \left[(1 + 2\sigma_z A) A^{-3\gamma} Z_{m1}^{-3\gamma} - \frac{2\sigma_z}{Z_{m1}} - 1 \right] Z_{m1}. \quad (21)$$

The damping factor, α_1, was determined from the relation (13).

The relative deviation, δ, was found from eq. (14) in the same manner as above (now N represents the values computed when the effect of the surface tension and viscosity was negligible, i.e. the large bubble sizes R_M, and M stands for values obtained for small bubble sizes, R_M). Calculated variations of δ_z, δ_p, δ_T, and δ_α, with the bubble size, R_M, are depicted in Fig. 4.

Finally, the limiting bubble sizes, R''_M, determined for $\delta = 5\%$ from Fig. 4, are displayed in Fig. 5.

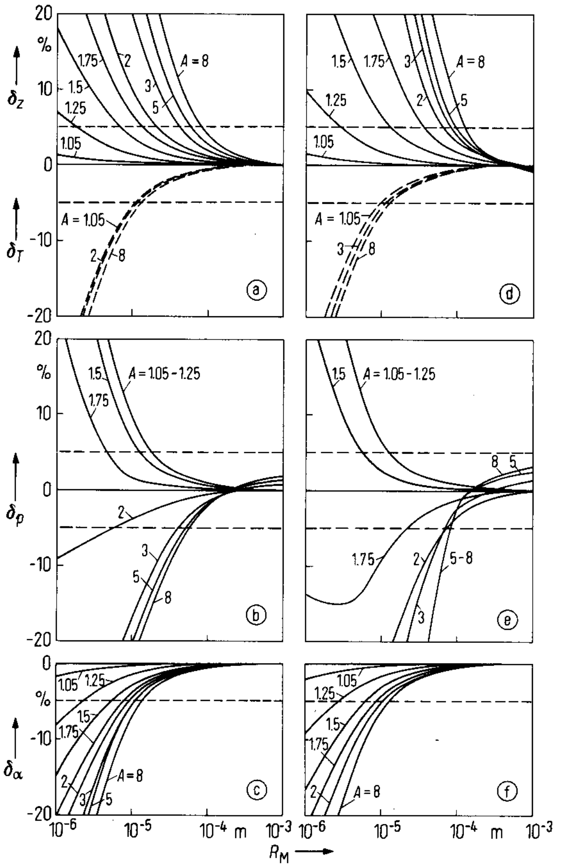

Fig. 4. Influence of the surface tension and viscosity (compression system, $\gamma = 1.4$):
(a) —— variation of the minimum bubble radius, Z_{m1}, with the bubble size, R_M, and the amplitude, A; --- variation of the compression time, T_{zc1}, with the bubble size, R_M, and the amplitude, A.
(b) Variation of the peak pressure, p_{zp1}, with the bubble size, R_M, and the amplitude, A.
(c) Variation of the damping factor, α_1, with the bubble size, R_M, and the amplitude, A.
Influence of the surface tension and viscosity (compression system, $\gamma = 1$):
(d) —— variation of the minimum bubble radius, Z_{m1}, with the bubble size, R_M, and the amplitude, A; --- variation of the compression time, T_{zc1}, with the bubble size, R_M, and the amplitude, A.
(e) Variation of the peak pressure, p_{zp1}, with the bubble size, R_M, and the amplitude, A.
(f) Variation of the damping factor, α_1, with the bubble size, R_M, and the amplitude, A.

3.3. *Influence of heat conduction*

Another factor influencing bubble behaviour is heat conduction and this is also the effect which is dependent on bubble size. However, because of great difficulties encountered in considering this factor properly we have not undertaken the computations of the respective limiting bubble sizes here. In the case of linear bubble oscillations appropriate data can be found, for example, in [13]. For non-linear bubble oscillations some computations have been performed, for example, by Hickling [14], Flynn [15], and Margulis and Dmitrieva [16]. Further references can be found in a recent review article by Prosperetti [17].

Let us note here that the effect of heat conduction also depends on the excitation technique. Thus, determination of the limiting bubble sizes should be done for each method separately.

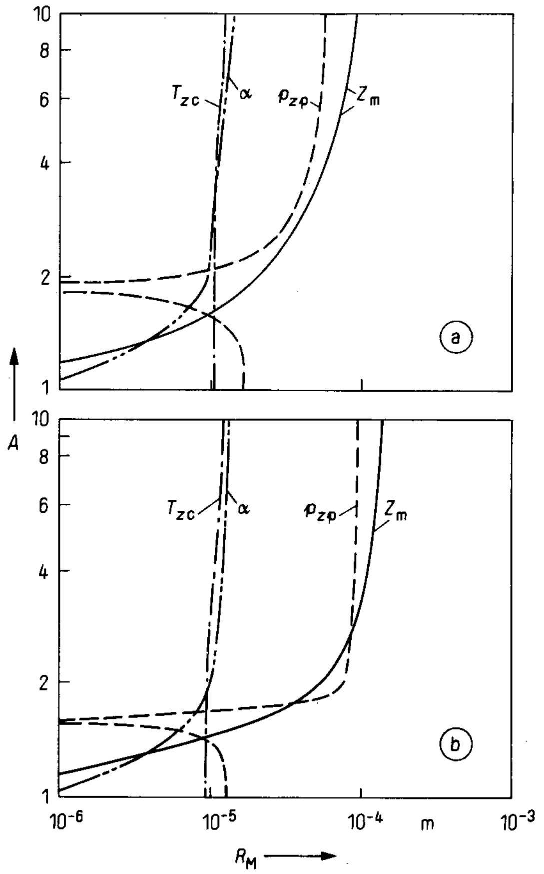

Fig. 5. Boundaries for the effects of the surface tension and viscosity (compression system, $\gamma = 1.4$ (a) and $\gamma = 1$ (b)).
—— minimum bubble radius, Z_m,
–·– compression time, T_{zc},
––– peak pressure, p_{zp},
–··– damping factor, α.

From the published data on free linear bubble oscillations (e.g. in [13]) it can be concluded that the heat losses are significant approximately for the range of bubble sizes $3\,\mu m \leqq R_e \leqq 3$ mm. However, as far as the non-linear bubble oscillations are concerned we do not know any theoretical work giving suitable data for determination of the limiting bubble sizes. Fortunately, as will be shown in the next section, we can estimate the limiting bubble size from experimental data published by Mellen [18].

4. Interpretation of Mellen's experiment

A measurement of the peak pressure, p_p, in the waves radiated by freely oscillating bubbles was reported by Mellen [18]. The measurement was done for a broad range of bubble sizes, R_M, which is important for tracing the size-dependent effects. Mellen's results are reproduced in Fig. 6 and we shall use them in determining the upper limiting bubble size for the effect of heat losses in the case of violently oscillating bubbles.

Mellen himself interpreted the measured data by inserting a straight line having a slope 3/2 (the broken line in Fig. 6) [18]. However, our interpretation is based on two straight lines a, b, also drawn in Fig. 6 (the full lines). Equations of the straight lines a, b are of the form $p_p = C_1 R_M$ and $p_p = C_2 R_M^k$, respectively. As follows from the first equation the constant C_1 can be interpreted as the non-dimensional peak pressure p_{zp} ($p_{zp} = p_p^* z$), multiplied by the ambient pressure p_∞. Let us note that in this case the pressure p_{zp} is independent of the bubble size, R_M, which is essential for our interpretation, because it indicates an absence of the size-dependent effects.

The second equation, on the other hand, represents a functional dependence of the pressure, p_{zp}, on the bubble size, R_M, this dependence being of the form $p_{zp} \sim R_M^{k-1}$. Though in a subsequent report [19] Mellen suggests an explanation for this drop in the peak pressure by a limited hydrophone re-

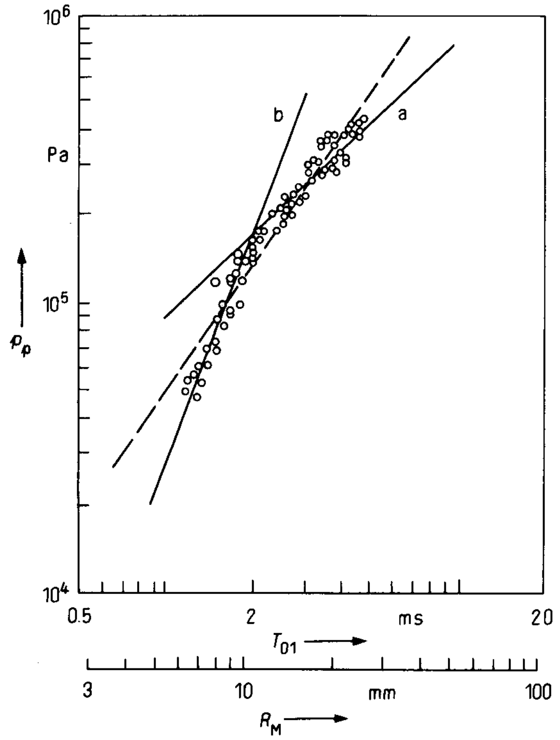

Fig. 6. Mellen's experimental data [18]: the peak pressure, p_p, at $r = 1$ m, vs. the time of bubble oscillation, T_{01}. The bubble size, R_M, is determined from the approximate formula $R_M = 0.543\, T_{01} (p_\infty/\varrho_\infty)^{1/2}$ [12, 18], where p_∞ = 100 kPa and $\varrho_\infty = 10^3$ kg m^{-3}.

sponse, our opinion is that the factor responsible for this size-dependence is the thermal loss. Our hypothesis is supported by analysis of the bubble thermal behaviour [13−17] unknown at the time the report [19] originated.

The two straight lines a and b cross each other approximately for $R_M = 10.9$ mm. Hence, in view of our interpretation, this point can be regarded as the lower limit, R_M''', of the non-linearly oscillating scaling bubbles, and this size is in quite good agreement with the findings of the linear theory, where, as discussed in Paragraph 3.3. it was determined that $R_e''' = 3$ mm. Note that $R_M \doteq R_e$ for linearly oscillating bubbles.

Two remarks should be made at this point. First, Mellen generated bubbles by underwater electric discharges (spark and exploding-wire tests). It is our opinion that the bubbles generated in this way are vaporous in nature (particularly the spark bubbles) and the value of R_M''' presented above is, strictly speaking, valid only for this kind of bubble. Second, even if the vaporous bubbles can be modelled to a first approximation by the gas bubbles [12], very little is known about the intensity of their oscillations (i.e. about A) at present. Thus we know one co-ordinate only, i.e. R_M''', and this is insufficient for positioning precisely the border-point in the bubble map.

5. Discussion

In our opinion the scaling law represents a convenient tool both for theoretical and experimental workers. By eliminating the bubble size from the basic equations it enables one to simplify theoretical analysis. In experiments it should enable one to produce a direct comparison of data obtained with bubbles of different sizes. The importance of these properties becomes more evident if we realize that for non-scaling bubbles a unique numerical solution and unique experimental data are obtained for each bubble size R_M.

However, because at present there are very few direct experimental proofs for the existence of the scaling law (except for Mellen's work [18] discussed in the previous section and some results reported in [6]), we should rather talk about the scaling property of the bubble models. Nevertheless, we hope that suitable experiments will be forthcoming to verify the present theories in a more or less restricted form.

The final bubble map showing the limits of influence for different parameters is displayed in Fig. 7. The borders have been drawn in such a way

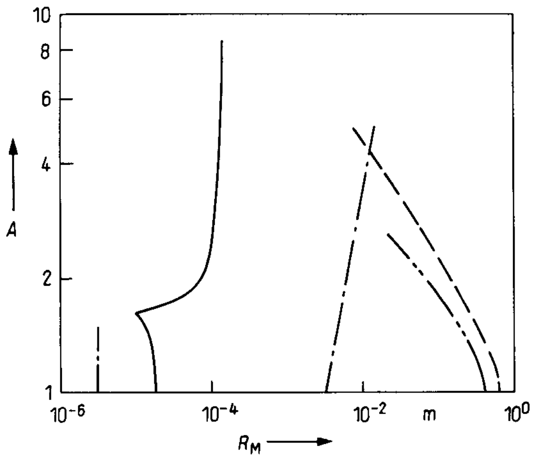

Fig. 7. The bubble map. Boundaries for the effects of:
--- gravity in the compression system,
–··– gravity in the expansion system,
–·– heat losses,
—— the surface tension and viscosity.

that in the inner region the significant values Z_m, p_{zp}, ... differ always by less than 5% from the values obtained when the size-dependent effects are disregarded. From Fig. 2 and Fig. 4 similar limits can also be easily determined for other values of δ. The limits were found for physical conditions given in Section 1. Unfortunately, for other conditions, unless the principle of similarity can be applied, the new limits have to be determined from eqs. (8) and (16) by new numerical computations.

It can be seen from Fig. 7 that the scaling bubbles occupy only a small region on the bubble map. However, it is our opinion that it is this very region that can be most important for gaining proper insight into the behaviour of bubbles.

On the other hand, the region, where heat losses must be considered, and hence where analysis is extremely difficult, is very extensive and spans more than 3 decades. Unfortunately, a very important class of bubbles, namely the cavitation bubbles, seem to belong just here.

It should be stressed that the bubble models used to determine the limits in Fig. 7 often only represent first approximations to the real situations. For example, a closer examination of Taylor's term reveals that translational energy thus defined increases the total bubble energy [12]. This is apparently not correct because the energy associated with the upward rise must come at the expense of the potential energy. (As can be verified, the problem is more complex than a simple change of the sign of the translational energy.)

Similarly, in determining the boundaries of influence for the surface tension and viscosity, the heat losses from the bubble interior were not considered. However, it is well-known (see, e.g. [15]) that heat losses play a very important role for bubbles of sizes $R_M \doteq 0.1$ mm and hence their omission from the analysis distorts the results in a barely predictable manner. In order to account somehow for the heat losses the calculations were done both with the adiabatic and isothermal models, representing the limiting cases of a bubble's thermal behaviour.

Finally, the limits of influence for the heat losses were estimated only from the results of linear theory [13] and from Mellen's measurements [18].

When examining the effects of the surface tension and viscosity, the computations were carried out for amplitudes as large as $A \leqq 8$. Computations of bubble oscillations for such large amplitudes are certainly problematic. However, as this question is briefly discussed elsewhere [11], we refer the reader to that work.

It may be seen in Figs. 4b and 4e that for $A \doteq 1.6 \ldots 1.9$ the quantity p_{zp} is almost independent of R_M. No closer examination of this phenomena has been undertaken here. In this connection, however, let us note that approximately for the same value of A the behaviour of the scaling bubbles was almost independent of γ [12].

As mentioned in Section 3 the limits of influence for the surface tension and viscosity should be computed for each excitation technique separately. The analysis based on the constant amplitude, A, as performed here, has an advantage in providing a unifying approach [10]. However, it has a drawback in postulating unequal excitation conditions for bubbles of different sizes. For example, it follows from eq. (20) that equality of amplitudes means in fact smaller values of P_m^* for smaller bubbles.

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