ON THE ASSUMPTION OF A UNIFORM PRESSURE FIELD IN THE BUBBLE INTERIOR

Karel Vokurka

1. INTRODUCTION

A common assumption taken in works on bubble dynamics is that the pressure field in the bubble interior is uniform [1 – 4]. Many authors do not even mention this assumption explicitly; nevertheless it is present in their equations implicitly.

In order that this assumption should not be violated, the bubble wall velocity is usually required to be much lower than the velocity of sound in the gas [4 – 6]. To this day, however, there is no work where this criterion has been evaluated quantitatively. The reason is, most probably, the seeming triviality of the problem.

As we shall try to show here, the relation between the wall velocity and the velocity of sound in the gas in the bubble interior is not as simple as it perhaps seems to be at first sight. For example, the velocity of sound in the gas depends on the gas temperature. However, both the wall velocity and the gas temperature vary with the wall position. A maximum wall velocity and the respective gas temperature grow with the intensity of the bubble oscillations. The final temperature is further determined by the initial temperature, the method of bubble excitation, the bubble size, and the physical properties of the gas and liquid. Similar arguments apply in the case of the wall velocity. Thus the mutual relation of the two velocities is influenced by a number of factors.

It is the aim of this paper to investigate the relationship between the two velocities mentioned in greater detail. For this purpose free radial oscillations of medium-sized spherical gas bubbles will be considered. As the effects of gravity, surface tension, viscosity, and heat conduction can be neglected for the medium-sized bubbles [4], the analysis is substantially simplified and this enables us to gain a better insight into the processes involved.

This work represents an elaboration of some ideas from Samek's paper on the collapse of a gas bubble in the liquid [6], recently published in this Journal.

2. A THEORETICAL MODEL

Let us consider a gas bubble excited in a certain way for free oscillations [7]. The bubble wall motion can be fairly exactly described using Gilmore's model [8]:

$$\ddot{R}R\left(1-\frac{\dot{R}}{C_l}\right)+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3C_l}\right)= \tag{1}$$

$$=H\left(1+\frac{\dot{R}}{C_l}\right)+\frac{R}{C_l}\dot{H}\left(1-\frac{\dot{R}}{C_l}\right),$$

where

$$C_l = c_{l\infty}\left(\frac{P+B}{p_\infty+B}\right)^{(n-1)/2n}, \tag{2}$$

and

$$H=\frac{1}{\varrho_{l\infty}}\frac{n}{n-1}(p_\infty+B)\left[\left(\frac{P+B}{p_\infty+B}\right)^{(n-1)/n}-1\right]. \tag{3}$$

Here R is the bubble wall radius, C_l and $c_{l\infty}$ are the velocities of sound in the liquid at the bubble wall and at infinity, respectively, and $\varrho_{l\infty}$ is the liquid density at infinity. P and p_∞ are the pressures in the liquid at the bubble wall and at infinity, respectively, H is the enthalpy difference between liquid at pressures P and p_∞, and finally B and n are constants in the Tait equation of state for the liquid. The dots denote differentiation with respect to time. The initial conditions of equation (1) are $R(0) = R_0$ and $\dot{R}(0) = \dot{R}_0 = 0$.

Let us now assume that the gas in the bubble interior behaves as a perfect gas and that there is no heat flow across the gas-liquid interface. Finally, the pressure and temperature fields in the bubble are assumed to be uniform. Then the pressure at the wall, P, which equals the pressure throughout the bubble, and the gas temperature, Θ, will vary during the bubble oscillations as

$$P = P_0\left(\frac{R}{R_0}\right)^{-3\gamma}, \tag{4}$$

$$\Theta = \Theta_0\left(\frac{R}{R_0}\right)^{-3(\gamma-1)}. \tag{5}$$

Here P_0 and Θ_0 are the initial (i.e. when $R = R_0$) pressure and temperature in the gas, respectively, and γ is the ratio of the specific heats.

The wall of an oscillating bubble represents a source of pressure disturbances: a diverging spherical wave is propagated from the wall outwards into the surrounding liquid and a converging spherical wave inwards into the gas in the bubble.

As long as the sound velocity in the gas is much higher than the wall velocity the effects of disturbances will have time to be distributed evenly throughout the gas and hence the pressure field in the bubble interior remains, as assumed, uniform [5].

The velocity of sound in a gas is defined as $c^2 = (\mathrm{d}p/\mathrm{d}\varrho)_a$ (see, e.g. [9]), where p and ϱ are gas pressure and density, respectively, and the subscript a refers to the adiabatic change associated with the wave propagation. If we denote the velocity of sound, when $\Theta = \Theta_0$, as c_0, it is possible readily to obtain a well-known result from the above definition formula [9]

$$c^2 = c_0^2\left(\frac{\Theta}{\Theta_0}\right). \tag{6}$$

With respect to equation (5) this can be further rearranged to give [6]

$$c^2 = c_0^2\left(\frac{R}{R_0}\right)^{-3(\gamma-1)}. \tag{7}$$

In equations (4) to (7) a number of parameters, such as P_0, Θ_0, and c_0, occur, whose values are essential for the present problem. It is the purpose of the next section to specify these parameters in accordance with known experiments.

3. THE INITIAL CONDITIONS

There are three basic techniques of exciting a bubble for free oscillations [7]. As the initial conditions and the values of the respective parameters are unambiguously determined by these techniques, they will be briefly described in the following.

3.1. Excitation by decreasing bubble energy (the compression system)

In this case $R_0 = R_M$, $P_0 = P_m$, $\Theta_0 = \Theta_m$, and $c_0 = c_m$. Here the subscripts M and m denote the maximum and minimum values, respectively.

This technique is experimentally implemented in two ways. First, a steep pressure change $\Delta p > 0$ (e.g. a shock wave) of relatively long duration ΔT (exceeding the bubble life) is propagated through a liquid containing a gas bubble [10]. Second, an auxiliary vessel containing a gas of pressure $P_m < p_\infty$ is suddenly smashed in the liquid [11–14].

In all these experiments the initial gas temperature was approximately equal to the liquid temperature, i.e. $\Theta_0 = \Theta_m = \Theta_\infty$, where Θ_∞ is the liquid temperature at infinity. The intensity of bubble oscillations can be experimentally varied through the change of the ratios $\Delta p/p_\infty$ or P_m/p_∞. However, it is our opinion that in the latter case there are serious limitations imposed on the resulting intensity of bubble oscillations by the presence of the vessel remnants.

3.2. Excitation by increasing bubble energy (the expansion system)

In this case $R_0 = R_m$, $P_0 = P_M$, $\Theta_0 = \Theta_M$, and $c_0 = c_M$.

This technique is also experimentally used in two ways. First, a compressed gas of pressure $P_M > p_\infty$ is either injected into the liquid [15] or a pressurized auxiliary vessel (e.g. a thin wall glass sphere containing gas of pressure $P_M > p_\infty$) is burst in the liquid [16]. The initial gas temperature in these experiments was approximately equal to the liquid temperature, i.e. $\Theta_0 = \Theta_M = \Theta_\infty$. The intensity of bubble oscillations can be varied through the change of the ratio P_M/p_∞.

The second technique is based on an exogenic reaction in an explosive, during which a gas at high pressure $P_M \gg p_\infty$ and temperature $\Theta_M \gg \Theta_\infty$ is generated [17]. The value of P_M depends on the chemical composition of the explosive. It can also be partially varied by changing the density of the given explosive [18]. Finally, the intensity of bubble oscillations can also be varied by changing the ambient pressure, p_∞, e.g. by firing charges at great depths in the ocean [19].

3.3. Excitation by a transient change of the ambient pressure (the equilibrium system)

In this case $R_0 = R_e$, $P_0 = P_e = p_\infty$, $\Theta_0 = \Theta_e$, and $c_0 = c_e$. Here the subscript e denotes the equilibrium values.

In experiments using this excitation technique the bubble is exposed to a transient change, Δp, of the ambient pressure. However, duration of the pressure change, ΔT, must be shorter than the bubble life. The sign of the pressure change can be both positive and negative. The excitation by a transient change of the ambient pressure was used, for example, in works [20–22], where short pressure pulses were employed.

In the case of excitation by transient change of the ambient pressure the intensity of bubble oscillations can be varied by changing both the magnitude of Δp and of ΔT. Again, the initial gas temperature in the experiments was equal to the temperature of the surrounding liquid, i.e. $\Theta_0 = \Theta_e = \Theta_\infty$.

4. A SAMPLE COMPUTATION

Equations (1) to (4) were normalized and solved both in the compression and expansion systems [4, 7]. The initial conditions for the two systems are given in Section 3. For the purpose of comparison of different excitation techniques, a non-linear amplitude, A, defined by a relation $A = R_M/R_e$ [7] is used here as a measure of bubble oscillation intensity.

The initial gas pressure in the compression system, P_m, is related to the amplitude, A, by a formula $P_m = p_\infty A^{-3\gamma}$. In the expansion system the initial gas pressure, P_M, is used to determine the equilibrium radius, R_e, from a relation $R_e = R_m(P_M/p_\infty)^{1/(3\gamma)}$.

Now, however, the maximum radius, R_M, must be determined by integrating the equation of motion (1).

Equations (1) to (4) were solved in both systems for a number of initial pressures, P_0, and for several values of the parameter γ. The integration was terminated in the first compression phase, at the moment the bubble wall velocity attained a maximum, $\dot{R}_{max}$. For each computer run the value of $\dot{R}_{max}$ and of the corresponding wall position, R_v, was recorded. Furthermore, in the expansion system the value of the first maximum wall radius, R_M, was also determined.

Computations were performed for water under ordinary laboratory conditions, i.e. $p_\infty = 100$ kPa, $\varrho_{l\infty} = 10^3$ kg m^{-3}, $c_{l\infty} = 1450$ m s^{-1}, B = 300 MPa, and n = 7.

From the calculated values of R_v the respective velocity of sound in the gas, c_v, was determined using equation (7).

An example of computed variations of $\dot{R}_{max}$ and c_v with A is shown in Fig. 1. The gas was assumed to be air ($\gamma = 1{\cdot}4$, $c_0 = 332$ m s^{-1}, $\Theta_\infty = 293$ K). In Fig. 1 the broken line a represents the variation of the velocity of sound, c_v, in the compression system, and the broken line b in the expansion system (pressurized "cold" air).

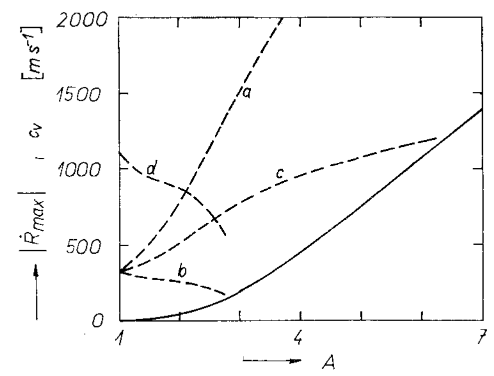

Fig. 1. Variation of the maximum bubble wall velocity, $\dot{R}_{max}$, and of the corresponding velocity of sound in the bubble interior, c_v, with the amplitude of oscillations, A: ——— the maximum bubble wall velocity $\dot{R}_{max}$, — — — — the velocity of sound in the gas c_v, (a — compression system, b — "cold" expansion system, c — equilibrium system, d — "hot" expansion system).

The broken line c corresponds to the equilibrium system, for which the ratio R_v/R_e can easily be determined from the results in the compression system [4]. Finally, the broken line d corresponds to the expansion of hot air ($\Theta_M = 3300$ K). This is to simulate the underwater explosions. However, because the physical properties of the gaseous explosion products (γ, c_0) differ from those of air, the conclusions drawn from Fig. 1 can only be approximate in this case.

Though analysis presented here does not provide direct evidence for the existence or non-existence of a uniform pressure field inside the bubble, it does provide some information on the limits of validity of the uniformity assumption. For example, it can be expected that when the wall velocity, $\dot{R}$, approaches the velocity of sound in the gas, c, a converging spherical shock is formed in the bubble [6, 23]. Propagation of this shock is accompanied by dissipation of energy and by steep pressure and temperature changes and thus the assumption is evidently not valid any more.

It can be seen from Fig. 1 that for moderate amplitudes of bubble oscillations the wall velocity is much lower than the velocity of sound in air. If the amplitude is increased, the maximum wall velocity, $\dot{R}_{max}$, and the corresponding velocity of sound in the gas, c_v, will approach each other and the two curves eventually cross for a certain amplitude A_c. The value of A_c depends on the properties of the gas (γ, c_0), the initial temperature Θ_0 and on the excitation technique. The larger γ, c_0, and Θ_0 are, the larger is A_c. From the point of view of the excitation technique, the excitation by decreasing bubble energy yields the largest A_c, and the excitation by increasing bubble energy the lowest A_c.

As has just been said, the maximum wall velocity approaches the velocity of sound in air first in the case of the "cold" expansion (the broken line b in Fig. 1), when $A_c = 2{\cdot}8$. However, as far as the actual experiments mentioned in Section 3 are concerned, it seems that the bubble wall velocities are always distinctly subsonic. For example, Heuckroth and Glass [16] worked with a maximum excess pressure $P_M/p_\infty = 36$, which corresponds to a theoretical amplitude $A = 1{\cdot}78$. Maksakov and Roi [15] used a maximum excess pressure $P_M/p_\infty = 150$, so that $A = 2{\cdot}05$. Both of these amplitudes are well below the amplitude $A_c = 2{\cdot}8$. A similar situation occurs in the case of underwater explosions ("hot" expansion), though the initial pressures are of the order $P_M = 10$ GPa [18] and hence theoretical amplitudes are as large as $A = 2{\cdot}84$ for $\gamma = 1{\cdot}4$ and $A = 2{\cdot}54$ for $\gamma = 1{\cdot}25$. Now, however, the higher initial temperatures are the cause of higher sound velocities, c_v, and hence also of higher amplitudes A_c.

In the case of the compression system the gas temperature and hence also the velocity of sound in the gas increase so fast that at first the distance between the two velocities with growing amplitude even increases (the two curves will also cross each other, but the respective point is outside the scope of Fig. 1). This means that the conditions for maintaining the uniform pressure field are even being improved with growing amplitude. This is a very interesting result that contradicts what one would intuitively expect.

Results presented in Fig. 1 were determined for air. A somewhat different picture would be obtained for other gases. For those having larger γ (e.g. triatomic gases) the wall velocity increases even more slowly and the gas temperature (and hence also the velocity of sound in the gas) even more quickly than in the case of air, which means better conditions for preserving the uniformity. However, the opposite is true for gases with $\gamma < 1{\cdot}4$, where the conditions for the formation of the converging shocks are thus more favourable. For example, in the case of hypothetical gases with $c_0 = 332\ \mathrm{m\,s^{-1}}$, $\gamma = 1{\cdot}25$, and $\gamma = 1{\cdot}33$ the values of A_c in the equilibrium system are $A_c = 4{\cdot}2$ and $A_c = 5{\cdot}3$, respectively.

When the basic equations were formulated in Section 2, the gas was assumed to be perfect. This assumption will be correct only for moderate amplitudes. For larger amplitudes a more realistic equation of state for the gas is necessary [24]. As follows from equation (6) the velocity of sound in a perfect gas is a function of temperature only. However, in a real gas (such as air) the velocity of sound grows both with the temperature and pressure. Thus, in fact, the actual value of c_v is even higher than that given by equation (7) and the uniformity of the pressure field should be preserved for even larger amplitudes than found here.

The spatial non-uniformity considered here was assumed to be a result of the violent bubble wall motion. However, as shown, for example, in [25], the non-uniform fields may also occur in bubbles performing forced oscillations at higher frequencies of the driving pressure field. In this case the amplitude of bubble oscillations may even be small.

The results presented can be used both in experimental and theoretical works. In experiments they can help to judge whether converging shocks are formed in the gas under given conditions. In theoretical studies they can help to determine the upper limits of validity for the assumption of uniformity. This is very important first of all in connection with compression and equilibrium systems, because there the parameters controlling the intensity of bubble oscillations can be selected in such a way as to obtain an arbitrarily large amplitude A. Then the values of amplitudes, A_c, computed here can be used as a quick, rough check of model validity.

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ON THE ASSUMPTION OF A UNIFORM PRESSURE FIELD IN THE BUBBLE INTERIOR

The relation between the bubble wall velocity and the velocity of sound in the bubble interior is investigated in greater detail. A sample calculation is performed for an air bubble in water and three basic excitation techniques. Using the criterion of the equality of the two velocities, the limits, for which the uniformity of the pressure field can be validly assumed, are evaluated quantitatively.

[Received April 1, 1985]

Ing. Karel Vokurka, CSc., Zdeňka Nejedlého 25, Jablonec n. N., CS-466 04, Czechoslovakia.