

ON RAYLEIGH'S MODEL OF A FREELY OSCILLATING BUBBLE. III. LIMITS

K. Vokurka*)

*Department of Physics, Faculty of Electrical Engineering,
Czech Technical University, Suchbátarova 2, 166 27 Praha 6, Czechoslovakia*

In the paper the range of bubble sizes for which the effects of gravity, surface tension, viscosity, and heat conduction can be neglected is determined. The equations of motion for macrobubbles and microbubbles are also presented.

1. INTRODUCTION

In the preceding papers [1, 2] equations of motion for the bubble wall were derived and solved for a broad range of amplitudes. However, until now the influences of gravity, surface tension, viscosity, and heat conduction have been omitted from the analysis. It will be shown in this paper that the omission is fully justified in the case of medium-sized bubbles, the actual dimensions of which will be therefore determined. Thus, in doing so, the limits of validity of the previous simple analysis will be found.

2. INFLUENCE OF GRAVITY

In the previous analysis the influence of gravity was deliberately omitted. This resulted in a model of a stationary bubble that did not vertically rise in the liquid. However, it is well known that real bubbles, when left free in the liquid, float upward (Archimedes' law). The upward velocity of the bubble, U , depends on the bubble size and it holds approximately that the larger the bubble, the larger is the velocity. As the velocity U does not grow linearly with R_M , it is possible to expect that for sufficiently small bubbles this vertical motion can be neglected to a first approximation. The aim of this section is to determine the size of bubbles for which the simple analysis neglecting gravity is still valid.

According to Taylor [3] the kinetic energy associated with a translational motion of a bubble, E_t , is given by

$$(III-1) \quad E_t = \frac{\pi}{3} \rho R^3 U^2,$$

where the upward velocity, U , equals

$$(III-2) \quad U = \frac{2g}{R^3} \int_0^t R^3 dt.$$

Here g is the acceleration of gravity.

*) Present address: *Department of Research and Development, LIAZ n.p., tř. V. Kopeckého 400, 466 05 Jablonec n. N., Czechoslovakia*

In this paper only a somewhat simplified approach based on the compression phase and on an assumption of a constant ambient pressure, p_∞ , will be followed.

The energy equation for the compression phase equals

$$(III-3) \quad \Delta E_p + E_t = E_k + E_i .$$

It is evident that as long as E_t is negligible in comparison with the full energy, E_{pM} , the simple previous analysis gives satisfactory results.

Let us note here that Taylor's analysis represents just a first approximation to the effect of gravity. This can be seen best from eq. (III-3), where the translational energy increases the total bubble energy. This is apparently not correct because the energy associated with the upward motion must come at the expense of the potential energy. However, as far as the first period of oscillations in the expansion system is concerned, Cole [3] reports a good agreement between calculated and observed bubble motions. It can then be expected that the accuracy of results regarding only the compression phase will also be satisfactory.

Let us pass to the nondimensional compression variables. Defining the nondimensional acceleration of gravity and upward velocity as $g_z = gR_M(\varrho/p_\infty)$ and $U^* = U(\varrho/p_\infty)^{1/2}$, respectively, we obtain

$$(III-4) \quad E_{zt} = \frac{1}{4}U^{*2}Z^3 ,$$

$$(III-5) \quad U^* = 2g_z \frac{1}{Z^3} \int_0^{\tau_z} Z^3 d\tau_z ,$$

$$(III-6) \quad \Delta E_{zp} + E_{zt} = E_{zk} + E_{zi} .$$

The limiting bubble size will be found in two ways: (1) from the energy relations given above, and (2) from the solution of the equation of motion. First, let us evaluate eqs. (III-4)–(III-6). If the upper limit of integration in (III-5) is set to be T_{zc} , eqs. (III-4) and (III-5) can be rewritten

$$(III-7) \quad E_{ztc} = \frac{1}{4}U_c^{*2}Z_m^3 ,$$

$$(III-8) \quad U_c^* = 2g_z \frac{1}{Z_m^3} \int_0^{T_{zc}} Z^3 d\tau_z .$$

The integral can be easily evaluated for $A \rightarrow 1$ and $\gamma = 4/3$. Then $Z = Z_m = 1$ and from (II-4) we find that $T_{zc} = \pi/2$. Hence $U_c^* = \pi g_z$ and $E_{ztc} = \pi^2 g_z^2 / 4$. Considering the ordinary values of p_∞ , ϱ and g [1] we have $g_z = 0.1 R_M$ and finally $E_{ztc} = 2.5 \times 10^{-3} \pi^2 R_M^2$. Supposing that the influence of gravity can be neglected for $E_{ztc} \leq 0.01$ we find the corresponding limiting radius to be $R'_M = 2/\pi \doteq 0.6$ m.

For other values of A the integral in (III-8) must be evaluated numerically. For the limiting case $A \rightarrow \infty$ the value of the integral, I , approximately equals $I \doteq 0.5$. It follows from fig. II-1 that this value of I can be used conveniently for any suffi-

ciently intensive oscillations. We then have $U_c^* = 0.1 R_M / Z_m^3$ and $E_{ztc} = 2.5 \times 10^{-3} R_M^2 Z_m^{-3}$. Substituting, as above, in this last relation the value $E_{ztc} \leq 0.01$ we finally obtain an approximate formula for the limiting maximum radius

$$(III-9) \quad R'_M = 2Z_m^{1.5}, \quad A \geq 2.$$

Thus the limiting bubble size depends on A : the larger the A , the smaller the R'_M .

The second approach in finding R'_M is based on the equation of motion. That can be determined by the usual procedure from eq. (III-6). We obtain

$$(III-10) \quad \ddot{Z}Z + \frac{3}{2}\dot{Z}^2 = A^{-3\gamma}Z^{-3\gamma} + \frac{U^{*2}}{4} - 1,$$

where U^* is given by eq. (III-5).

Eqs. (III-5) and (III-10) were solved for a number of amplitudes A and sizes R_M . The initial conditions were $Z(0) = 1$ and $\dot{Z}(0) = 0$. From computed values of Z_m and p_{zp} (T_{zc} varied very little with R_M) a relative deviation, δ , was calculated using the formula

$$(III-11) \quad \delta = \frac{M - N}{N} \cdot 100 \quad [\%].$$

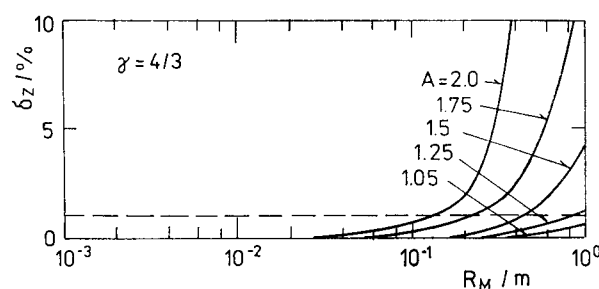


Fig. III-1. Influence of gravity: change of Z_m with growing R_M .

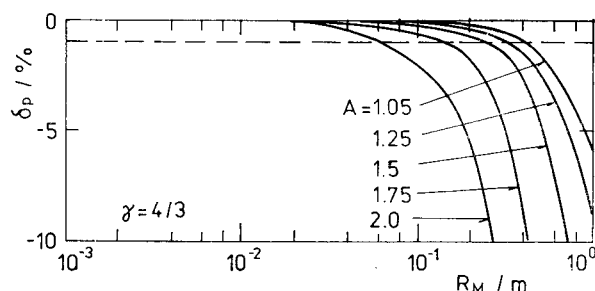


Fig. III-2. Influence of gravity: change of p_{zp} with growing R_M .

Here M means the values of Z_m and p_{zp} obtained from eq. (III-10) and N the values obtained from eq. (I-12). Dependences of δ_z and δ_p on R_M are depicted in figs. III-1 and III-2.

For a given deviation, δ , it is possible to obtain the dependence of the limiting radius, R'_M , on A from figs. III-1 and III-2. The curves obtained for $\delta_z = \delta_p = 1\%$

are displayed in fig. III-3. It follows from fig. III-3 that Rayleigh's model, which does not consider the influence of gravity, gives satisfactory results for bubbles with the maximum radius $R_M \leq R'_M \doteq 0.3$ m ($A \leq 1.5$) and $R_M \leq R'_M \doteq 0.1$ m ($1.5 < A \leq 2$). It can also be seen that for bubbles oscillating with $A > 2$ the limiting radius, R'_M , will be even smaller than 0.1 m, a fact that also follows from (III-9).

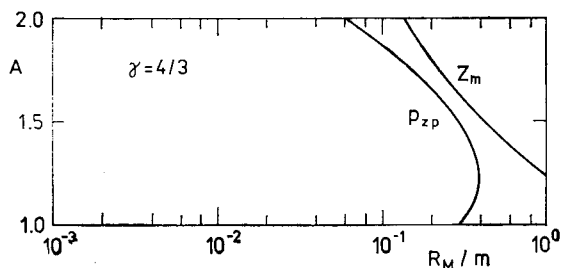


Fig. III-3. Limiting bubble sizes for $\delta_Z = \delta_p = 1\%$.

Bubbles larger than the limiting size will be called macrobubbles. They are most often generated by underwater explosions and will be not treated in this study. A more detailed account regarding them can be found in [3].

The plane "size-amplitude" ($R_M - A$) is very instructive in studying the behaviour of bubbles with different sizes and amplitudes. It will be referred to as the bubble map.

3. INFLUENCE OF THE SURFACE TENSION

Further factors not included in the previous analysis are the surface tension, viscosity, and heat conduction. It will be shown in this and in the following sections that these factors are important only for sufficiently small bubbles. As in the preceding section, we shall therefore attempt to find the limiting size of the bubbles, R''_M , for which these factors can still be neglected to a reasonable degree.

The surface energy, E_s , associated with the bubble wall is given by the relation

$$(III-12) \quad E_s = 4\pi\sigma R^2,$$

where σ is the surface tension constant. The surface energy has a maximum, E_{sM} , for $R = R_M$. When the bubble wall moves from R_M to R , the decrease in the surface energy, ΔE_s , equals

$$(III-13) \quad \Delta E_s = 4\pi\sigma(R_M^2 - R^2).$$

The surface tension tends to contract the bubble and thus the energy relation for the compression phase is

$$(III-14) \quad \Delta E_p + \Delta E_s = E_k + E_i.$$

At this point we shall pass to the nondimensional quantities. Defining the non-

dimensional surface tension constant as $\sigma_z = \sigma/(p_\infty R_M)$, eqs. (III-13) and (III-14) can be rewritten to give

$$(III-15) \quad \Delta E_{zs} = 3\sigma_z(1 - Z^2),$$

$$(III-16) \quad \Delta E_{zp} + \Delta E_{zs} = E_{zk} + E_{zi}.$$

In eq. (III-16) both the potential and the surface energies represent driving energies in the bubble compression. Let us therefore evaluate how their mutual share varies during the wall motion from Z_M to Z . From (I-9) and (III-15) we have

$$(III-17) \quad \frac{\Delta E_{zs}}{\Delta E_{zp}} = 3\sigma_z \frac{1 - Z^2}{1 - Z^3},$$

which equals $2\sigma_z$ for $Z \rightarrow 1$ and $3\sigma_z$ for $Z \rightarrow 0$. It follows then that the relative share of the surface energy as a driving energy increases from $2\sigma_z \Delta E_{zp}$ at the beginning of the compression to $3\sigma_z \Delta E_{zp}$ at the end of the contraction. Hence, the role of the surface tension increases with advancing compression; however, this increase is not decisive.

From (III-15) the maximum of the surface energy is $E_{zsM} = 3\sigma_z$. It is evident that as long as E_{zsM} can be neglected in comparison with E_{zPM} the simple analysis of the previous papers [1, 2] will yield satisfactory results. Using the definition of σ_z this limit can easily be found. If it is required that $E_{zsM} \leq 0.01$, then it follows that R_M must satisfy the relation

$$(III-18) \quad R_M \geq R_M'' = \frac{3\sigma}{p_\infty} \times 10^2,$$

and, as we can see, the limiting size, R_M'' , does not depend on A .

For example, for ordinary values of σ and p_∞ [1] we find that the influence of the surface tension can be neglected for bubbles with the maximum radius $R_M \geq R_M'' \doteq \pm 0.23$ mm.

The equation of motion can be found by the usual procedure from eq. (III-16). Using (I-7)–(I-9) and (III-15) we obtain

$$(III-19) \quad \ddot{Z}Z + \frac{3}{2}\dot{Z}^2 = P_m^* Z^{-3\gamma} - \frac{2\sigma_z}{Z} - 1.$$

In this equation P_m^* represents the measure of the oscillation intensity. It is possible to use also nonlinear amplitude, A , for this purpose; however, now the simple relation (II-22) does not hold any more. In the presence of the surface tension the pressure in the liquid at the bubble wall, when $R = R_e$, is

$$(III-20) \quad P_e = p_\infty + \frac{2\sigma}{R_e}.$$

From (I-1) the pressure in the gas, when $R = R_e$, equals

$$(III-21) \quad P_e = P_m \left(\frac{R_M}{R_e} \right)^{3\gamma}.$$

Equating (III-20) with (III-21) and using the definition formula for the amplitude ($A = R_M/R_e$) the relation between P_m^* and A can be found in the form

$$(III-22) \quad P_m^* = (1 + 2\sigma_z A) A^{-3\gamma}.$$

It follows from (III-20) that owing to the surface tension the equilibrium pressure exceeds the pressure p_∞ , when $R = R_e$. However, it can be verified that the kinetic energy of the liquid still has a maximum, E_{kmax} , at the equilibrium radius.

It will be shown in the next sections that for the range of bubble sizes where the surface tension becomes important, viscosity and heat conduction must also be considered. As all these influences act together, there is no sense in solving the equation of motion for the surface tension alone and we therefore postpone this task until the other two effects are taken into account.

4. INFLUENCE OF VISCOSITY

Viscosity represents the only damping mechanism that is currently considered in connection with Rayleigh's model. The energy dissipation due to viscosity, E_v , was found by Poritsky [4] to be

$$(III-23) \quad E_v = 16\pi\eta \int_0^t R\dot{R}^2 dt,$$

where η is the coefficient of dynamic viscosity.

Now the energy relation has the form

$$(III-24) \quad \Delta E_p = E_k + E_i + E_v.$$

Introducing nondimensional viscosity, $\eta_z = \eta/[R_M(\rho p_\infty)^{1/2}]$, we obtain a nondimensional form of (III-23) and (III-24)

$$(III-25) \quad E_{zv} = 12\eta_z \int_0^{\tau_z} Z\dot{Z}^2 d\tau_z,$$

$$(III-26) \quad \Delta E_{zp} = E_{zk} + E_{zi} + E_{zv}.$$

Again, let us first determine the limiting size of bubbles for which viscosity can be neglected, from energy consideration. For this purpose the integral in (III-25) must be evaluated. As in section 2, the upper limit of integration will be set to be T_{zc} .

The integral can be easily solved in the case of linear oscillations. In the X system eq. (III-25) has the form

$$(III-27) \quad E_{xvc} = 12\eta_x \int_0^{T_{xc}} \dot{X}^2 d\tau_x,$$

where $\eta_x = \eta/[R_e(\rho p_\infty)^{1/2}]$. Using (II-2)–(II-4), the value of the integral in (III-27)

is $I = (\pi/2)(3\gamma)^{1/2} A_x^2$. For $E_{xvc} \leq 0.01$ and $\gamma = 1$ eq. (III-27) can be rearranged to give

$$(III-28) \quad R_e \geq R_e'' = 3.3 \times 10^{-4} A_x^2.$$

Thus in the case of linear oscillations the limiting bubble size, R_e'' , grows as the square of the amplitude A_x . For example, if $A_x = 0.05$, then viscosity can be approximately neglected for bubbles with an equilibrium radius $R_e \geq 0.8 \mu\text{m}$, and for $A_x = 0.1$ for bubbles with $R_e \geq 3.3 \mu\text{m}$.

In the case of nonlinear oscillations the integral must be computed numerically. For example, if $A = 2$ and $\gamma = 4/3$, we obtain that $I \doteq 0.65$. Then for E_{zvc} to be negligible in (III-26) (i.e. $E_{zvc} \leq 0.01$), we find from (III-25) that $R_M \geq R_M'' = 78 \mu\text{m}$. The value of the integral, I , was obtained in a model without surface tension and heat conduction. Since for bubbles of this size (i.e. $R_M \doteq 0.1 \text{ mm}$) both these effects are important, this result must be considered with certain caution.

It follows from the foregoing discussion that the effect of viscosity depends on the amplitude A . The greater the amplitude, the larger the limiting size of bubbles R_M'' . It is reasonable to expect that for sufficiently large amplitudes viscosity will become important even for bubbles with $R_M > 0.1 \text{ mm}$. Nevertheless, for amplitudes $A \leq 2$ viscosity is important even for still smaller bubbles than is the case with the surface tension. Therefore eq. (III-26) has to be extended to include also the surface energy.

We obtain

$$(III-29) \quad \Delta E_{zp} + \Delta E_{zs} = E_{zk} + E_{zi} + E_{zv}.$$

Using (I-7)–(I-9), (III-15) and (III-25) the equation of motion can be determined in the form

$$(III-30) \quad \ddot{Z}Z + \frac{3}{2}\dot{Z}^2 = P_m^* Z^{-3\gamma} - \frac{2\sigma_z}{Z} - 4\eta_z \frac{\dot{Z}}{Z} - 1.$$

Some results obtained with eq. (III-30) will be given in the next section, after the last factor, heat conduction, has been briefly discussed.

5. INFLUENCE OF HEAT CONDUCTION

So far it has been supposed that the bubble behaves as a thermally isolated system, i.e. that its compressions and expansions are adiabatic. This assumption is based on the relatively low heat conductivity of gases. However, numerous theoretical and experimental studies have shown (e.g., [5–8]) that if the bubble size is being decreased, starting from a certain radius, R_M'' , the heat transfer between the gas and the liquid becomes more and more important, and there will be a transition from adiabatic to isothermal behaviour. This transition finds a place for the size of bubbles, R_M , approximately between 3 mm and 3 μm . The bubbles with $R_M \geq 3 \text{ mm}$ behave as an adiabatic system, and the bubbles with $R_M \leq 3 \mu\text{m}$ as an isothermal one. In the transition region, i.e. for $3 \mu\text{m} < R_M < 3 \text{ mm}$, the thermal behaviour of the

bubbles is rather complicated. There is a partial heat transfer across the interface accompanied by heat losses. There is also a change in the thermal behaviour during the wall motion from R_M to R_m . Whereas in the vicinity of R_M the motion is basically isothermal, in the vicinity of R_m , where the wall velocity is very high, it is adiabatic. In the transition region heat conduction becomes, if not predominant, then at least one of the most important dissipative mechanisms [5, 7, 8]. Inclusion of the heat losses into the equation of motion presents a difficult task and will be omitted here. As a first approximation, thermal behaviour of bubbles in the transition region will be considered to be polytropic. Unfortunately, since the polytropic change is a reversible one, such a model does not account for the heat losses.

As can be easily verified the equation of motion based on the assumption of the polytropic change will have the same form as eq. (III-30), the only change being in replacing the adiabatic exponent γ with a polytropic exponent n , where $1 < n < \gamma$. The value of n to be used in eq. (III-30) depends on R_M . In the vicinity of $R_M = 3 \text{ mm}$ it approximately holds that $n \doteq \gamma$. With decreasing R_M , n also decreases, until in the vicinity of $R_M = 3 \text{ }\mu\text{m}$, $n \doteq 1$.

If the bubble behaves as the isothermal system ($R_M \leq 3 \text{ }\mu\text{m}$) the work done on the gas during the compression is given by the equation

$$(III-31) \quad E_i = \frac{4}{3}\pi P_m R_M^3 \ln(R_M/R)^3.$$

The nondimensional form of (III-31) is

$$(III-32) \quad E_{zi} = P_m^* \ln(1/Z)^3.$$

Substituting (III-32) into (III-29), we may obtain, in the usual way, the equation of motion for the bubble wall. It may be verified that even in the isothermal case the equation of motion has the same form as (III-30); however, now $\gamma = 1$. Thus eq. (III-30) represents a universal one because it is valid not only for the adiabatic model, but also for the polytropic and isothermal ones, if only the correct values of γ or n are used.

The significant values Z_m and T_{zc} can be determined by numerical integration of eq. (III-30). To determine the pressure p_{zp} , eq. (II-23) must be modified. Since the peak pressure at the wave is radiated when $Z = Z_m$ and $\dot{Z} = 0$, eqs. (I-25) and (III-30) can be arranged to give

$$(III-33) \quad p_{zp} = Z_m \left[(1 + 2\sigma_z A) A^{-3\gamma} Z_m^{-3\gamma} - \frac{2\sigma_z}{Z_m} - 1 \right].$$

Eq. (III-30) was numerically integrated for a number of amplitudes A , bubble sizes R_M , and for values of $\gamma = 1$ and $\gamma = 4/3$. The computed values of Z_m were also used to determine p_{zp} from eq. (III-33). Then, using relation (III-11) the relative deviations δ_z , δ_T , and δ_p were found. Now the solutions of eqs. (III-30) and (III-33) were substituted for M and solutions of eqs. (I-12) and (II-23) for N . Dependences of δ_z , δ_T , and δ_p on R_M thus found are given in figs. III-4 to III-7.

To determine the limits of validity of eq. (I-12), the values of R_M'' corresponding to $\delta = 1\%$ and to a given amplitude, A , were determined from figs. III-4 to III-7. The results thus obtained are displayed in figs. III-8 and III-9.

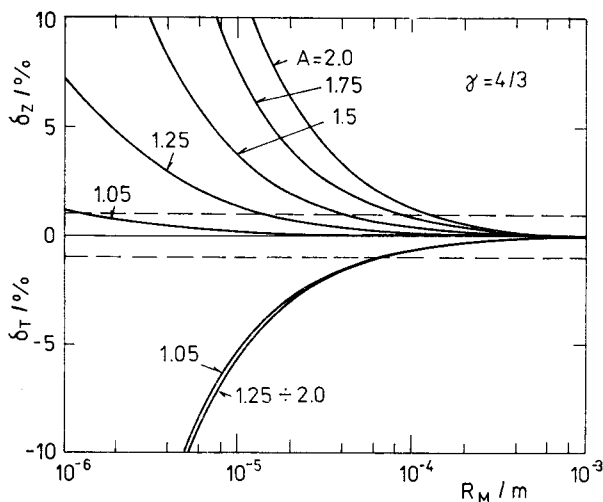


Fig. III-4. Influence of surface tension and viscosity: change of Z_m and T_{zc} with decreasing R_M . $\gamma = 4/3$.

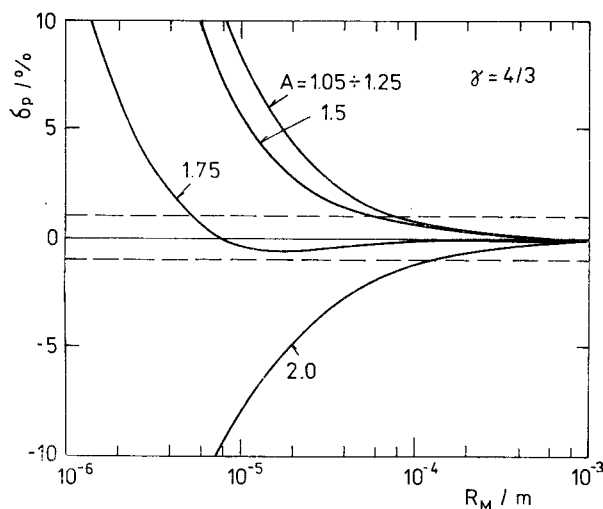


Fig. III-5. Influence of surface tension and viscosity: change of p_{zp} with decreasing R_M . $\gamma = 4/3$.

It can be concluded from figs. III-8 and III-9 that as far as the surface tension and viscosity are concerned the simple model (I-12) gives satisfactory results for $R_M \geq R_M'' \approx 0.1 \text{ mm}$ ($A < 2$). However, eq. (III-30) does not include the heat losses. As said above, these become significant for bubbles with $R_M < 3 \text{ mm}$. Thus the lower limit of the simple Rayleigh model (I-12) can be approximately given as $R_M'' \approx 3 \text{ mm}$. Bubbles with a radius in the range of microns, i.e. with $R_M < 1 \text{ mm}$, will be called microbubbles.

Before closing this section let us derive an equation of motion for linearly oscillating microbubbles. Passing in (III-30) from the Z to the X variables and linearizing the resulting equation we easily obtain

$$(III-34) \quad \ddot{X} + 4\eta_x \dot{X} + [3\gamma + (3\gamma - 1) 2\sigma_x] X = 0.$$

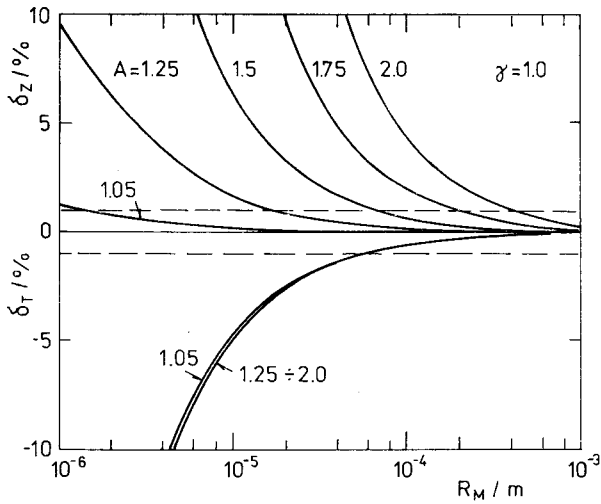


Fig. III-6. Influence of surface tension and viscosity: change of Z_m and T_{zc} with decreasing R_M . $\gamma = 1.00$.

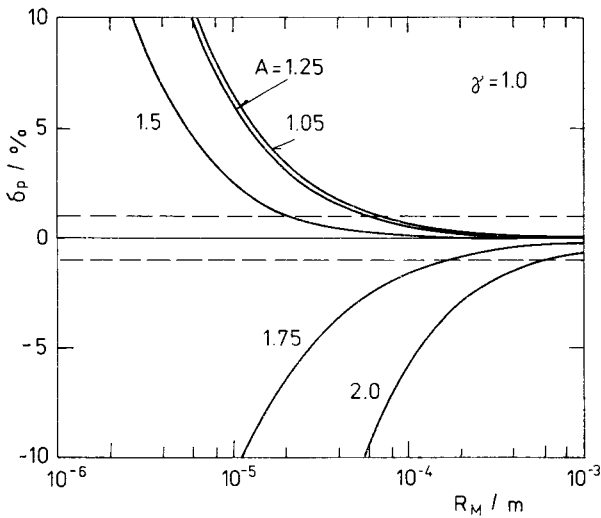


Fig. III-7. Influence of surface tension and viscosity: change of p_{zp} with decreasing R_M . $\gamma = 1.00$.

Here the nondimensional surface tension, σ_x , is defined by the relation $\sigma_x = \sigma / (R_e p_\infty)$.

Eq. (III-34) describes damped free linear oscillations of a microbubble. As discussed above the value of γ has to be selected according to the bubble size. The solution of eq. (III-34) for the initial conditions $X(0) = A_x$ and $\dot{X}(0) = 0$ has the

well-known form

$$(III-35) \quad X = A_x \exp(-\delta_x \tau_x) \cos(\omega_{xd} \tau_x).$$

Here, the nondimensional circular frequency of damped oscillations, ω_{xd} , is given by the relation

$$(III-36) \quad \omega_{xd}^2 = \omega_{xo}^2 - \delta_x^2,$$

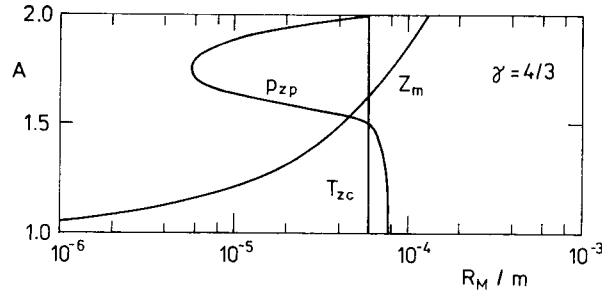


Fig. III-8. Limiting bubble sizes for $\delta_Z = \delta_p = \delta_T = 1\%$. $\gamma = 4/3$.

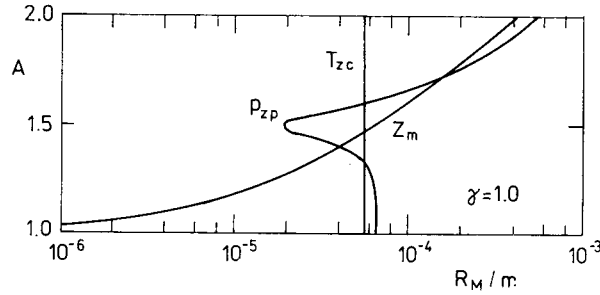


Fig. III-9. Limiting bubble sizes for $\delta_Z = \delta_p = \delta_T = 1\%$. $\gamma = 1.00$.

where the nondimensional circular frequency of undamped free oscillations, ω_{xo} , equals

$$(III-37) \quad \omega_{xo}^2 = 3\gamma + (3\gamma - 1) 2\sigma_x,$$

and the nondimensional damping constant, δ_x , ($\delta_x = \delta R_c(q/p_\infty)^{1/2}$) is

$$(III-38) \quad 2\delta_x = 4\eta_x.$$

6. CONCLUSION

As has been shown in the paper there is a certain range of bubble sizes, for which satisfactory results can be obtained without taking into account the effects of gravity, surface tension, viscosity, and heat conduction. For amplitudes $A \leq 2$ this range spans approximately two decades from $R'_M = 0.3$ m to $R''_M = 3$ mm. Rayleigh's model of these medium-sized bubbles is rather simple, and, what is even more important, the solution of the equation of motion does not depend on the bubble size. Thus, there are only two parameters involved in the equation of motion, namely the amplitude A and the adiabatic exponent γ . However, this is not true about the macrobubbles or microbubbles, where a unique solution is obtained for each R_M .

Fortunately, at least some of the bubbles used in laboratory experiments seem to belong in this medium size range [9–12], though further work is needed to clarify the limits for amplitudes $A > 2$.

For macrobubbles the effect of gravity becomes important and appropriate models can be found in [3]. On the other hand, for microbubbles the surface tension, viscosity, and heat conduction cannot be neglected. The analysis is especially complicated in the transition region (i.e. when R_M is between 3 mm and 3 μm), where heat losses significantly influence the bubble motion.

Though there is a vast literature on bubblelogy the author believes that there are still too many unsolved basic problems in bubble dynamics. For example, the existing experimental works (see e.g. [3, 12, 13]) show serious discrepancies between calculated (the calculations are meant in models considering the liquid compressibility) and measured energy losses during bubble pulsations. This indicates that the existing theoretical models do not take into account all important factors.

It is our opinion that the bubbles should be studied primarily in their simplest form. As has been shown in the paper, this simplest form is represented just by the free oscillations of the medium-sized gas bubbles. Only when this case is thoroughly explored and the respective theoretical models give reliable predictions, should the microbubbles or forced nonlinear oscillations be studied in greater depth. Otherwise we run the risk of erroneous theoretical conclusions which are almost impossible to rebut by experiments for the present.

Another factor affecting the complexity of the bubble behaviour is the amplitude. It is clear that the larger the amplitude, the more complicated the processes involved in the bubble motion. For the reasons mentioned above it seems therefore natural to start the research first with the small amplitude oscillations, and to advance to more violent oscillations only when this area has been fully mastered. Unfortunately, as far as we know, no systematic experimental research in this direction has been undertaken.

Received 5. 4. 1984.

References

- [1] Vokurka K.: Czech. J. Phys. B 35 (1985) 28.
- [2] Vokurka K.: Czech. J. Phys. B 35 (1985) 110.
- [3] Cole R. H.: Underwater Explosions, Princeton University Press, Princeton, 1948.
- [4] Poritsky H.: in Proc. of the 1st U.S. National Congress on Appl. Mech., (ed. E. Sternberg), New York, 1952, p. 813.
- [5] Devin Ch., Jr.: J. Acoust. Soc. Am. 31 (1959) 1654.
- [6] Hickling R.: J. Acoust. Soc. Am. 35 (1963) 967.
- [7] Flynn H. G.: J. Acoust. Soc. Am. 57 (1975) 1379.
- [8] Flynn H. G.: J. Acoust. Soc. Am. 58 (1975) 1160.
- [9] Mellen R. H.: J. Acoust. Soc. Am. 28 (1956) 447.
- [10] Lauterborn W.: Acustica 31 (1974) 51.
- [11] Shima A., Tomita Y.: Ing.-Arch. 51 (1981) 243.
- [12] Vokurka K.: in The 7th Conf. of Czechosl. Physicists, Praha, 1981, paper 09–07 (in Czech).
- [13] Hentschel W., Lauterborn W.: Appl. Sci. Res. 38 (1982) 225.