

## EXCITATION OF GAS BUBBLES FOR FREE OSCILLATIONS BY INCREASING THEIR ENERGY

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A method for the excitation of gas bubbles into free oscillations based on increasing their energy is analyzed and it is shown that the intensities of the bubble oscillations attainable with this technique are rather limited. This stands in contradiction to an approach often found in literature when in theoretical computations large intensities are considered in connection with this technique.

### 1. INTRODUCTION

As discussed in reference [1] gas bubbles can be excited for free oscillations either by decreasing bubble energy, by increasing bubble energy, or by transient change of the ambient pressure. In this paper the second method is to be considered in greater detail.

Excitation by increasing bubble energy can be achieved experimentally in several ways. First, a submerged auxiliary vessel (e.g., a thin walled glass sphere) can be pressurized to a pressure higher than the ambient pressure in the liquid and then burst, thus initiating the bubble growth and subsequent oscillations [2]. In the case of underwater explosions the gas is produced at a very high initial pressure and temperature as a result of the exothermic reaction in an explosive [3]. A further method incorporates a jet through which the compressed gas is injected into the liquid [4–7]. Finally, a method based on a sudden decrease of the ambient pressure in the liquid also falls into this category. However, in order that the bubble can remain a gas bubble (the case considered here) the ambient pressure should not drop under the liquid vapour pressure.

The objectives of this paper are, first, to give a brief analysis of this excitation method, second, to give graphs of two basic scaling functions, and finally to show that in contrast to excitation by decreasing bubble energy, where in theory any value of the bubble oscillation intensity can be achieved, the intensities attainable by this method are rather limited.

Some results regarding this excitation method have also been presented in reference [1]. However, because of the assumption of liquid non-compressibility that particular analysis was limited to the region of low initial pressures only. Here, as the assumption of liquid non-compressibility is removed, the analysis can be extended to cover much higher initial pressures.

To simplify the analysis, only spherical, medium-sized bubbles, for which the effects of gravity, surface tension, viscosity, and heat conduction can be neglected [8], will be considered here.

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## 2. EXCITATION OF GAS BUBBLES BY INCREASING THEIR ENERGY

Let a gas bubble of an initial minimum radius  $R_{m0}$  be at rest for  $t < 0$ . At  $t = 0$  the pressure inside the bubble (and thus also the internal energy of the gas) is instantaneously increased to a value  $P_{M0} > p_\infty$ , where  $p_\infty$  is the ambient pressure in the liquid for  $t < 0$ . Due to the excess pressure,  $P_{M0} - p_\infty$ , the bubble will expand to a maximum radius,  $R_{M1}$ , and then perform several damped oscillations around its equilibrium radius  $R_e$ .

For further work it is convenient to introduce non-dimensional variables. In this paper use will be made of the expansion system of variables ( $W$ ), in which the non-dimensional time, bubble radius, pressure at the bubble wall, and pressure in the liquid at a point  $r$  are defined, respectively, as [1, 8, 9]

$$t_w = t / [R_{m0}(\rho_\infty / p_\infty)^{1/2}], \quad W = R / R_{m0}, \quad P^* = P / p_\infty, \quad p_w = (p / p_\infty - 1)r / R_{m0}.$$

Here  $\rho_\infty$  is the density of the undisturbed liquid.

An example of the calculated bubble wall time history in the expansion system is given in Figure 1.

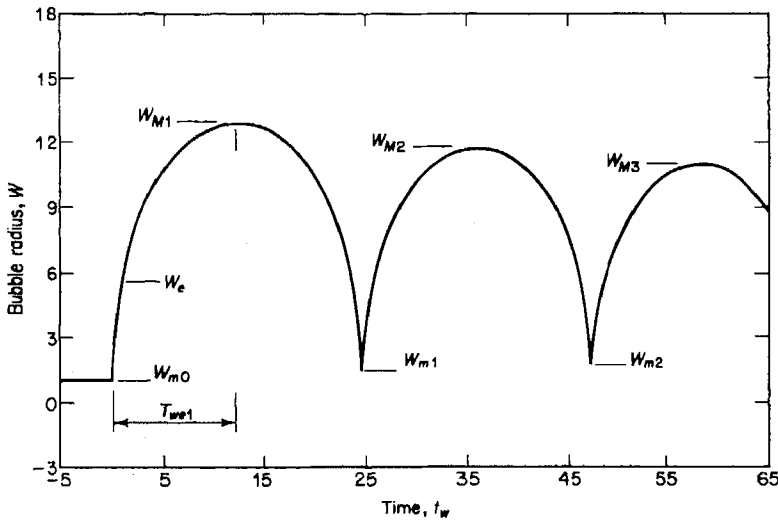


Figure 1. Bubble wall time history in the expansion system.  $P_{M0}^* = 10^3$ ,  $\gamma = 4/3$ ,  $p_\infty = 100$  kPa.

The motion of the bubble wall is accompanied by a pressure change,  $p$ , in the surrounding liquid. An example of the radiated wave calculated in the expansion system and for  $r \gg R_{m0}$  is given in Figure 2.

As can be seen in Figure 2, in the expansion system, after the process of bubble excitation has been initiated, a shock wave is radiated first. Later when the wall of the oscillating bubble is near minimum radii, waves of compression are emitted, and in the vicinity of maximum radii the bubble radiates waves of rarefaction. The waves of compression are usually referred to as the bubble pulses [3, 9].

The subscripts  $p$  and  $v$  in Figure 2 denote the peak and valley pressures, respectively. Note also that throughout this work the upper case letters refer to the bubble wall and the lower case letters to the points outside the wall.

The curves displayed in Figures 1 and 2 were calculated by using Gilmore's model (see Appendix A for the definition) and on the assumption of an ideal gas in the bubble

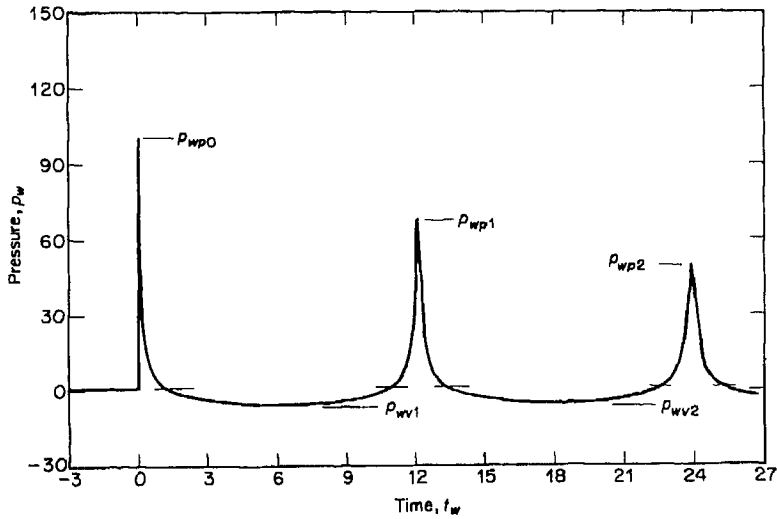


Figure 2. Radiated wave in the expansion system.  $P_{M0}^* = 10^2$ ,  $\gamma = 4/3$ ,  $p_\infty = 100$  kPa. Time  $t_w$  is measured from the moment of the shock wave arrival at a point  $r$ .

interior that undergoes adiabatic changes. In such a case the pressure at the bubble wall,  $P^*$ , varies as [1, 9]

$$P^* = P_{M0}^* W^{-3\gamma}, \tag{1}$$

where  $\gamma$  is the ratio of the specific heats.

A very important feature of the curves displayed in Figures 1 and 2 is that subsequent bubble radii minima and pressure maxima become smaller: i.e., (see also a discussion in reference [10])

$$W_{m0} < W_{m1} < W_{m2} < \dots, \quad p_{wp0} > p_{wp1} > p_{wp2} > \dots \tag{2, 3}$$

It should be noted here that the inequalities (2) and (3) are typical for the gas bubbles

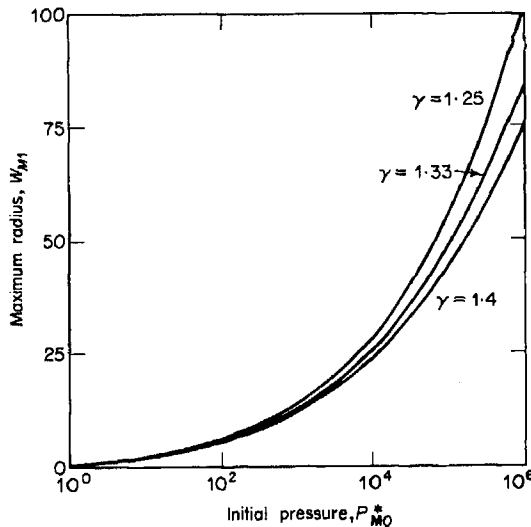


Figure 3. Variation of the first bubble maximum radius,  $W_{M1}$ , with the initial maximum pressure,  $P_{M0}^*$ . ( $p_\infty = 100$  kPa.)

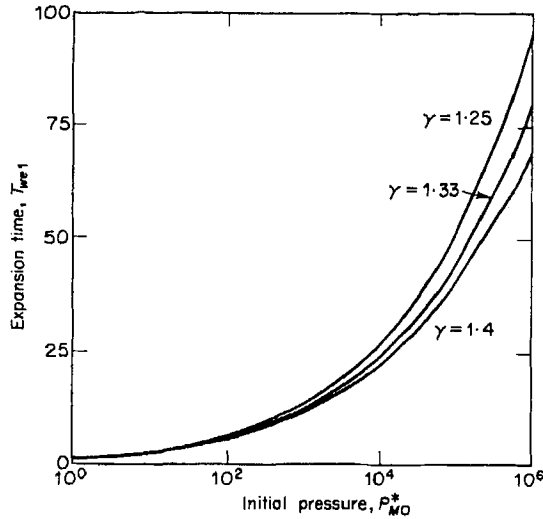


Figure 4. Variation of the first bubble expansion time,  $T_{we1}$ , with the initial maximum pressure,  $P_{M0}^*$ . ( $p_\infty = 100$  kPa.)

only and do not hold in general for vapour bubbles. Hence, as will be discussed elsewhere, they can be used to discriminate between the two kinds of bubbles in experiments.

Evidently, the higher the initial pressure,  $P_{M0}^*$ , the larger is the first maximum bubble radius,  $W_{M1}$ , and the longer it takes the bubble to expand to  $W_{M1}$ . The variations of  $W_{M1}$  with  $P_{M0}^*$ , and of the first expansion time,  $T_{we1}$ , with  $P_{M0}^*$ , calculated with the use of Gilmore's model and equation (1) are given in Figures 3 and 4. For  $P_{M0}^* \leq 10^2$  and  $\gamma = 4/3$  these functional dependences were also given in reference [1].

The functions  $W_{M1} = W_{M1}(P_{M0}^*, \gamma)$ , and  $T_{we1} = T_{we1}(P_{M0}^*, \gamma)$ , displayed in Figures 3 and 4, are of a certain type of scaling functions [11]. As will be shown elsewhere they can be used in the analysis of bubble behaviour and for evaluation of experimental data. Their counterparts in the compression system ( $Z$ ) together with examples of application were given in reference [11].

The curves displayed in Figures 3 and 4 were calculated for  $P_{M0}^*$  as large as  $10^6$ . Since ordinary gases behave as ideal ones only for pressures lower than 0.1–1 GPa, the higher pressure results (approximately for  $P_{M0}^* > 10^4$ ) given in Figures 3 and 4 should be regarded as a first approximation only.

### 3. AMPLITUDE OF BUBBLE OSCILLATIONS

To describe the intensity of the bubble oscillations, the amplitude,  $A$ , defined by

$$A = R_M / R_e = W_M / W_e, \quad (4)$$

was introduced in references [1, 9]. In the case of larger bubble sizes, for which the surface tension is unimportant [8], the equilibrium radius  $W_e$  can be determined from the condition that  $P^* = 1$  when  $W = W_e$  [9]. Then, in the case of the ideal gas one has from equation (1) that

$$W_e = (P_{M0}^*)^{1/(3\gamma)}, \quad (5)$$

and the corresponding value of  $W_{M1}$  can be taken from Figure 3.

Variation of the amplitude of the first oscillation,  $A_1 = W_{M1} / W_e$ , with  $P_{M0}^*$  is shown in Figure 5. The curves in Figure 5 were calculated under the same assumptions as those

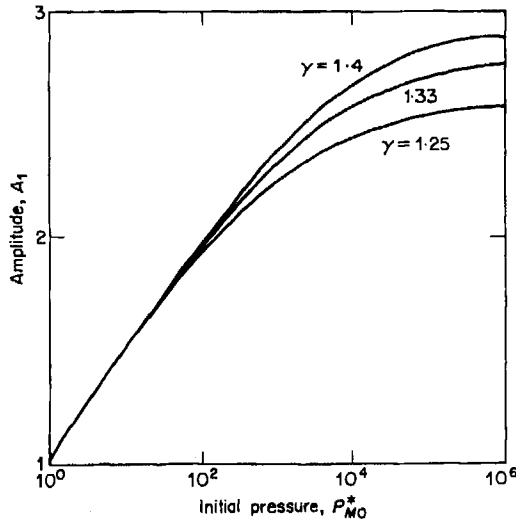


Figure 5. Variation of the first oscillation amplitude,  $A_1$ , with the initial maximum pressure,  $P_{M0}^*$ . ( $p_\infty = 100$  kPa.)

in Figures 3 and 4. However, because the amplitude is determined as the ratio of the wall positions  $W_{M1}$  and  $W_e$ , which correspond to low pressures  $P^*$ , this quantity is rather insensitive to the conditions occurring in the bubble at the initial moments (e.g., whether the gas behaves as ideal or not). Therefore these curves are particularly reliable even for the higher initial pressures  $P_{M0}^*$ .

An interesting feature worth noticing in Figure 5 is that the amplitude,  $A_1$ , in contrast to  $W_{M1}$ , does not grow appreciably for  $P_{M0}^* > 10^4$ . This decrease in the rate of the amplitude growth in the expansion system should be compared with the compression system  $Z$ , where [1, 9]

$$A_1 = (P_{m1}^*)^{-1/(3\gamma)}. \tag{6}$$

Here  $P_{m1}^*$  is the initial minimum pressure in the bubble in the compression system; i.e., when  $R = R_{M1}$ .

As follows from equation (6), by a suitable choice of  $P_{m1}^*$ , at least in theory, any amplitude  $A_1$  can be obtained. Thus, with respect to  $A_1$ , there is a distinct difference between the bubbles starting to oscillate from either  $R_{M1}$  or  $R_{m0}$ , or, in other words, between the compression and expansion systems.

The slow growth of the amplitude,  $A_1$ , with the pressure  $P_{M0}^*$  in the expansion system is a consequence of both the acoustical radiation in the vicinity of  $W_{m0}$  and the inherent non-symmetry in the bubble wall motion [1]. For example, as shown in Appendix B, even in Rayleigh's model, in which there is no radiation damping at all, the amplitude grows rather slowly with  $P_{M0}^*$ ; namely

$$A_1 = [(P_{M0}^*)^{(\gamma-1)/(3\gamma)}]/(\gamma-1)^{1/3}, \quad P_{M0}^* \geq 50. \tag{7}$$

As a concrete example one can consider the case when  $\gamma = 4/3$ . Then, according to equation (7), in the expansion system the amplitude  $A_1$  grows with  $P_{M0}^*$  only as the 12th root. On the other hand, in the compression system the amplitude  $A_1$  grows with  $P_{m1}^*$  as the 4th root (cf. equation (6)). The presence of the radiation damping then only amplifies the effect caused by the wall motion non-symmetry.

This theoretical limitation on the maximum attainable bubble oscillation amplitude in the expansion system agrees well with experimental data [11], where it was found that even in the case of bubbles generated by underwater explosions, when the initial pressures  $P_{M0}$  reach extremely high values (of the order of 10 GPa), the bubble oscillates only with a moderate amplitude  $A_1 < 2.5$ . Also to be noted here is that similar limitations would be obtained if other measures of oscillation intensity, such as  $P_{m1}^*$ ,  $P_{M1}^*$ , etc. [1], were used.

The result just obtained has an important consequence. Whereas in the compression system, when decreasing the initial pressure  $P_{m1}^*$  (and thus increasing  $A_1$ ), the pressure at the first bubble contraction,  $P_{M1}^*$ , markedly grows, only moderate values of  $P_{M1}^*$  can be obtained in the expansion system when increasing  $P_{M0}^*$ . For example, from the scaling functions given in reference [11] it follows that for amplitudes  $A_1 < 2.5$  attainable in the expansion system the pressures  $P_{M1}^*$  cannot be higher than  $P_{M1}^* = 700$  ( $\gamma = 1.25$ ,  $p_\infty = 100$  kPa). On the other hand, much higher pressures  $P_{M1}^*$  can be obtained in the compression system.

Similarly, whereas in the compression system the damping factor  $\alpha_1 = R_{M2}/R_{M1}$  can attain any low value if only a suitably low initial pressure  $P_{m1}^*$  is chosen, in the expansion system, because of the limitation on the maximum amplitudes attainable, the theoretical damping factor cannot drop under  $\alpha_1 = 0.88$  [11].

What has just been said is true only for dissipative models. In Rayleigh's model  $P_{M0}^* = P_{M1}^*$  and  $\alpha_1 = 1$  irrespective of the value of  $A_1$ . There are also experimental restrictions that must be taken into account when considering realistic values of the quantities mentioned.

#### 4. CONCLUSION

The most important result given in this paper is that gas bubbles excited for free oscillations by increasing their energy can oscillate only with relatively moderate amplitudes  $A_1 < 2.5$ . This is not so for the gas bubbles excited by decreasing their energy, when, at least in theory, much larger amplitudes can be achieved.

Thus it is not possible to interchange the two systems arbitrarily and, when beginning computations of the bubble wall motion at the first bubble maximum radius  $R_{M1}$ , whereas the bubble under consideration is excited by increasing its energy, care must be taken to use the correct value of the amplitude (or, which is equivalent, of  $P_{m1}^*$ ).

This requirement has not always been met in the past. For example, to match the energy dissipations of explosion and laser generated bubbles, respectively, Keller and Kolodner [12], Ebeling [13], and Keller [14] used amplitudes  $A_1 = 5$ ,  $A_1 = 9$ , and  $A_1 = 5$ , respectively. As has been shown in this paper, these amplitudes are unobtainable in the expansion system (and this system, after all, was the one the authors had in mind). The use of the excessive amplitudes may even, in a certain sense, be dangerous, as attribution of all the dissipative losses to the acoustical radiation may produce an illusory impression of the problem being solved. However, as was shown in reference [11], the problem is more complex, and there are some further causes of the energy losses that must be taken into account if correct predictions of the bubble behaviour are to be made.

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#### APPENDIX A: GILMORE'S MODEL

In Gilmore's model the equation of motion for the bubble wall has in the expansion system the form (for the dimensional form see, e.g., reference [11])

$$\ddot{W}W(1 - \dot{W}/C^*) + \frac{3}{2}\dot{W}^2(1 - \frac{1}{3}\dot{W}/C^*) = H^*(1 + \dot{W}/C^*) + (W/C^*)\dot{H}^*(1 - \dot{W}/C^*), \quad (\text{A1})$$

where the speed of sound in the liquid at the bubble wall,  $C^*$ , is

$$C^* = c_\infty^*[(P^* + B^*)/(1 + B^*)]^{(n-1)/(2n)}, \quad (\text{A2})$$

and the enthalpy change between pressures  $P^*$  and  $p_\infty^* = 1$  is given by

$$H^* = [n/(n-1)](1 + B^*)\{[(P^* + B^*)/(1 + B^*)]^{(n-1)/n} - 1\}. \quad (\text{A3})$$

Here  $c_\infty^* = c_\infty(\rho_\infty/p_\infty)^{1/2}$  is the speed of sound in the undisturbed liquid, and  $B$  and  $n$  are constants in the Tait equation of state for the liquid ( $B^* = B/p_\infty$ ). For water, which is considered in this work, the physical constants are  $c_\infty = 1450 \text{ ms}^{-1}$ ,  $B = 300 \text{ MPa}$ ,  $n = 7$ , and  $\rho_\infty = 10^3 \text{ kg m}^{-3}$ . The dots in equation (A1) denote differentiation with respect to time.

For moderate values of  $P_{M0}$  ( $P_{M0} < 10 \text{ MPa}$ ) the pressure in the liquid can be computed from a simple relation [9]: i.e.,

$$p_w = W(P^* - 1 + \frac{1}{2}\dot{W}^2), \quad r \gg R_{m0}. \quad (\text{A4})$$

#### APPENDIX B: MAXIMUM AMPLITUDES IN RAYLEIGH'S MODEL

The energy relation in the expansion system can be written as [1, 9]

$$\Delta E_{wi} = E_{wk} + \Delta E_{wp} + \Delta E_{wd}, \quad (\text{B1})$$

where the change of the internal energy of the ideal gas is

$$\Delta E_{wi} = P_{M0}^*(\gamma - 1)^{-1}[1 - W^{-3(\gamma-1)}]. \quad (\text{B2})$$

The kinetic energy of the non-compressible liquid is

$$E_{wk} = \frac{3}{2}\dot{W}^2 W^3, \quad (\text{B3})$$

and the change in the potential energy of the liquid is

$$\Delta E_{wp} = W^3 - 1. \quad (\text{B4})$$

$\Delta E_{wd}$  in equation (B1) is the dissipated energy. For medium-sized bubbles in the non-compressible liquid one assumes that  $\Delta E_{wd} = 0$ .

When  $W = W_M$  the kinetic energy is  $E_{wk} = 0$ . For  $P_{M0}^* \gg 1$  then also  $W_M^3 \gg 1$  and therefore the expression for potential energy (B4) can be simplified. Substituting equation (B2) and the simplified equation (B4) into relation (B1), taking further into account that  $P_{M0}^* = W_e^{3\gamma}$  and  $A = W_M / W_e$ , one obtains after some manipulation that

$$P_{M0}^* = [(\gamma - 1 + A^{-3\gamma})A^3]^{\gamma/(\gamma-1)}. \quad (\text{B5})$$

For the pressures considered, i.e., for  $P_{M0}^* \gg 1$ , the term  $A^{-3\gamma}$  can be neglected and one finally obtains

$$A \doteq [(P_{M0}^*)^{(\gamma-1)/(3\gamma)}] / (\gamma - 1)^{1/3}. \quad (\text{B6})$$