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EXCITATION OF GAS BUBBLES FOR FREE OSCILLATIONS

K. VOKURKA†

Department of Physics, Faculty of Electrical Engineering, Czech Technical University, Suchbátarova 2, CS-166 27 Praha 6, Czechoslovakia

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Methods for excitation of gas bubbles into free oscillations are classified and discussed. The analysis is based on Rayleigh's model of a medium-sized bubble. A non-linear amplitude is selected to be a universal measure of bubble oscillation intensity and its relation to natural intensity measures is determined.

1. INTRODUCTION

In the majority of theoretical works on the non-linear free oscillations of gas bubbles the wall motion is supposed to start at the maximum radius (see, e.g., references [1–6]). This approach is probably due to Rayleigh's original formulation of the problem [7]. Another reason for this is that the most important phenomena associated with intensive bubble oscillations occur during the final stages of compression and early stages of expansion and hence earlier phases are omitted from the analysis. As far as the bubble wall motion is concerned the respective initial conditions are well known ($R = R_M$, $\dot{R} = 0$) and hence no difficulties arise. However, this need not be true for some other variables, as, for example, the gas temperature, the value of which depends on previous bubble history. Some authors (see, e.g., references [1, 5, 6]) have defined the initial gas temperature to be equal to the temperature of the surrounding liquid. However, as will be shown here, such an approach may be justified only in a rather special case, when the bubble is excited by decreasing its energy.

To determine the initial temperatures in other cases, it is necessary to consider the way the bubble oscillation was excited. Strangely enough, little attention has been paid to this question in the literature on bubbles.

The aim of this paper is to classify and examine different excitation techniques. To gain a better insight into the processes involved, a bubble model as simple as possible will be used here. This simplest model is Rayleigh's model of a medium-sized spherical gas bubble [8], for which the effects of gravity, surface tension, viscosity, and heat conduction can be neglected, and a non-compressible liquid is assumed. Further, to simplify the analysis substantially, it is assumed that changes of the ambient pressure are in the form of step or rectangular pulses.

All numerical solutions presented here have been obtained with the standard Runge–Kutta method.

2. EXCITATION TECHNIQUES

There are three basic ways of exciting bubbles for free oscillations. These are (1) by decreasing bubble energy, (2) by increasing bubble energy, and (3) by transient change

† Present address Department of Research and Development, LIAZ n.p., tř. V. Kopeckého 400, CS-466 05 Jablonec n.N., Czechoslovakia.

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of the ambient pressure. Though corresponding equations of the bubble wall motion are known, they will be briefly derived here for the sake of completeness and clarity.

2.1. EXCITATION BY DECREASING BUBBLE ENERGY

Consider a gas bubble of radius R_M and let the bubble be initially at rest. At the moment $t=0$ the pressure of the gas in the bubble interior is instantaneously decreased to a value $P_m < p_\infty$, where p_∞ is the pressure in the liquid at infinity. Due to the excess pressure, $p_\infty - P_m$, the bubble will first contract to a minimum radius R_m and then oscillate between R_m and R_M. During the contraction the gas pressure, P, is supposed to increase according to the adiabatic law $P = P_m(R_M/R)^{3\gamma}$. Here γ is the ratio of the specific heats. The work done on the gas during the compression from R_M to R is

$$\Delta E_i = \tfrac{4}{3}\pi[1/(\gamma-1)]P_m R_M^3[(R_M/R)^{3(\gamma-1)} - 1].$$

The liquid, which was initially at rest, acquires a kinetic energy $E_k = 2\pi\rho\dot{R}^2R^3$, where ρ is the liquid density. The initial potential energy of the bubble was $E_{pM} = (4/3)\pi p_\infty R_M^3$. During the contraction from R_M to R the decrease of the potential energy equals

$$\Delta E_p = E_{pM} - E_p = \tfrac{4}{3}\pi p_\infty(R_M^3 - R^3).$$

Finally, the bubble compression is governed by the energy relation $\Delta E_p = E_k + \Delta E_i$.

At this point it is convenient to introduce the following non-dimensional quantities: time $t_z = t/[R_M(\rho/p_\infty)^{1/2}]$, radius $Z = R/R_M$, minimum gas pressure $P_m^* = P_m/p_\infty$, and energy $E_z = E/E_{pM}$. As these variables are introduced in connection with bubble compression they will be called the compression variables and the resulting system of equations the compression or Z system. The non-dimensional form of the relations given above is

$$\Delta E_{zi} = [1/(\gamma-1)]P_m^*[Z^{-3(\gamma-1)} - 1], \tag{1}$$

$$E_{zk} = \tfrac{3}{2}\dot{Z}^2Z^3, \qquad \Delta E_{zp} = 1 - Z^3, \qquad \Delta E_{zp} = E_{zk} + \Delta E_{zi}. \tag{2-4}$$

By substituting equations (1)-(3) into equation (4) and differentiating the resulting equation with respect to time one obtains

$$\ddot{Z}Z + \tfrac{3}{2}\dot{Z}^2 = P_m^* Z^{-3\gamma} - 1. \tag{5}$$

The initial conditions for this equation are $Z(0) = Z_M = 1$ and $\dot{Z}(0) = 0$. An example of the solution of equation (5) is given in Figure 1.

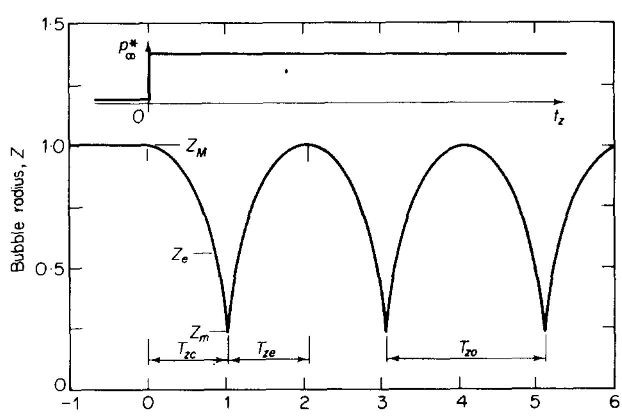

Figure 1. Excitation of a bubble by decreasing its energy: time history of the wall motion in the compression system. $P_m^* = 0{\cdot}1$, $\gamma = 4/3$. T_{zc}, T_{ze}, T_{zo}—times of bubble compression, expansion, and oscillation. Z_M, Z_e, Z_m—maximum, equilibrium, and minimum bubble radii.

In theory, the initial pressure reduction in the bubble interior can be achieved by an extremely fast evacuation of a portion of the gas from the bubble or by a violent cooling of the gas. However, as far as the author is aware, none of these techniques has been used experimentally. Another method is based on an instantaneous increase of the ambient pressure from p_∞ to p'_∞. In this case $P^*_m = p_\infty/p'_\infty$. However, for the same bubble size and liquid density as in the previous case, the actual period of oscillations, T_o, will decrease by a factor $(p_\infty/p'_\infty)^{1/2}$. The pressure jump from p_∞ to p'_∞ is also displayed in Figure 1. The last method, which has occasionally been used in experiments [9-12], involves an auxiliary vessel (e.g., a thin wall sphere), which can be partially evacuated. The vessel is then destroyed in the liquid, and thus an implosion is triggered. However, because of the vessel remnants the implosion can suffer some rather unpredictable disturbances.

2.2. EXCITATION BY INCREASING BUBBLE ENERGY

Let a gas bubble of radius R_m be initially at rest. At the moment $t = 0$ the pressure inside the bubble is instantaneously increased to a value $P_M > p_\infty$. This increase in the gas pressure will result in the bubble's expansion. During this expansion the gas pressure is supposed to decrease according to the adiabatic law $P = P_M(R_m/R)^{3\gamma}$. When the wall moves from R_m to R the work done by the gas equals

$$\Delta E'_i = \tfrac{4}{3}\pi[1/(\gamma-1)]P_M R_m^3[1-(R_m/R)^{3(\gamma-1)}].$$

At the same time the potential energy of the bubble increases from $E_{pm} = (4/3)\pi p_\infty R_m^3$ to E_p so that

$$\Delta E'_p = E_p - E_{pm} = \tfrac{4}{3}\pi p_\infty(R^3 - R_m^3).$$

The relation for the kinetic energy of the liquid was given above. Finally, the respective energy equation for the bubble expansion is $\Delta E'_i = E_k + \Delta E'_p$.

Introducing the non-dimensional expansion variables

$$t_w = t/[R_m(\rho/p_\infty)^{1/2}], \qquad W = R/R_m, \qquad P^*_M = P_M/p_\infty, \qquad E_w = E/E_{pm}$$

one obtains the following expansion (or W) system of equations:

$$\Delta E'_{wi} = [1/(\gamma-1)]P^*_M[1 - W^{-3(\gamma-1)}], \tag{6}$$

$$E_{wk} = \tfrac{3}{2}\dot{W}^2 W^3, \qquad \Delta E'_{wp} = W^3 - 1, \qquad \Delta E'_{wi} = E_{wk} + \Delta E'_{wp}. \tag{7-9}$$

Substitution of equations (6)-(8) into equation (9) and differentiation with respect to time gives

$$\ddot{W}W + \tfrac{3}{2}\dot{W}^2 = P^*_M W^{-3\gamma} - 1, \tag{10}$$

where the initial conditions are $W(0) = W_m = 1$ and $\dot{W}(0) = 0$. An example of the solution of equation (10) is given in Figure 2.

Excitation by increasing bubble energy may be achieved in several ways. One possible method is based on injecting a compressed gas into the liquid. Such a technique was used, for example, in the work described in references [13-15]. (Heuckroth and Glass [14] used thin wall glass spheres containing pressurized gas. This technique is just the opposite to the evacuated thin wall glass spheres mentioned in the preceding section.) Another method is based on a violent heating of the gas in the bubble interior, which thus increases the gas pressure. Such an excitation occurs, for example, in the case of underwater explosions, where the exothermic reaction in an explosive produces gases having high pressure and temperature [16]. A third possible method is based on an instantaneous lowering of the pressure in the liquid from p_∞ to p'_∞. In this case $P^*_M = p_\infty/p'_\infty$;

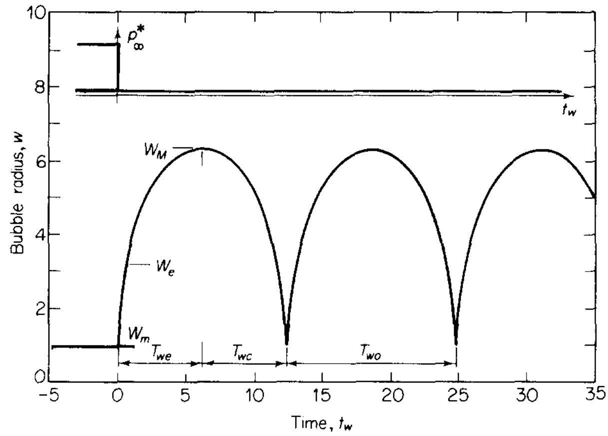

Figure 2. Excitation by increasing bubble energy: time history of the wall motion in the expansion system. $P^*_M = 100$, $\gamma = 4/3$. T_{we}, T_{wc}, T_{wo}—times of bubble expansion, compression, and oscillation. W_M, W_e, W_m—maximum, equilibrium, and minimum bubble radii.

however, the period of oscillations, T_o, will be increased by a factor $(p_\infty/p'_\infty)^{1/2}$. The pressure jump from p_∞ to p'_∞ is also displayed in Figure 2.

2.3. EXCITATION BY A TRANSIENT CHANGE OF THE AMBIENT PRESSURE

Let a bubble of radius R_e be initially at rest. At the moment $t = 0$ the pressure in the liquid at infinity is instantaneously changed to a new value $p'_\infty = p_\infty + \Delta p$, kept at this value for a time ΔT, and then returned to the original value p_∞. The pressure jump Δp may have a positive or negative sign and the time interval ΔT may have a value $0 < \Delta T < \infty$.

If $\Delta p > 0$, the bubble will contract first and the internal energy will vary as

$$\Delta E_i = \tfrac{4}{3}\pi[1/(\gamma-1)]p_\infty R_e^3[(R_e/R)^{3(\gamma-1)} - 1].$$

For $t < 0$ the potential energy of the bubble was $E_{pe} = (4/3)\pi p_\infty R_e^3$, and at $t = 0$ it increases to $E_{pM} = (4/3)\pi(p_\infty + \Delta p)R_e^3$; hence the change in the potential energy during the bubble contraction is

$$\Delta E_p = E_{pM} - E_p = \tfrac{4}{3}\pi(p_\infty + \Delta p)(R_e^3 - R^3).$$

The kinetic energy and the energy relation are the same as those given in section 2.1.

Introducing non-dimensional equilibrium variables

$$t_y = t/[R_e(\rho/p_\infty)^{1/2}], \qquad Y = R/R_e, \qquad \Delta p^* = \Delta p/p_\infty, \qquad E_y = E/E_{pe}, \tag{11}$$

one obtains the equilibrium (or Y) systems of equations

$$E_{yk} = \tfrac{3}{2}\dot{Y}^2 Y^3, \qquad \Delta E_{yp} = (1 + \Delta p^*)(1 - Y^3),$$
$$\Delta E_{yi} = [1/(\gamma-1)][Y^{-3(\gamma-1)} - 1], \qquad \Delta E_{yp} = E_{yk} + \Delta E_{yi}. \tag{12-14}$$

By the usual procedure one derives the equation

$$\ddot{Y}Y + \tfrac{3}{2}\dot{Y}^2 = Y^{-3\gamma} - (1 + \Delta p^*), \tag{15}$$

with the initial conditions $Y(0) = Y_e = 1$, $\dot{Y}(0) = 0$.

If $\Delta p < 0$ the bubble will expand first. With the same non-dimensional Y variables as above it is easy to see that now $\Delta E'_{yi} = -\Delta E_{yi}$, $\Delta E'_{yp} = -\Delta E_{yp}$, and finally $\Delta E'_{yi} = E_{yk} + \Delta E'_{yp}$. By the usual procedure one derives equation (15) again.

There are basically two ways of changing the ambient pressure temporarily. First, the bubble is stationary and a pressure disturbance (a wave of compression, rarefaction, or a tension wave) travels through the liquid. Second, a liquid containing the bubble flows through a region where the pressure is either increased or decreased.

According to the value of ΔT_y ($\Delta T_y = \Delta T/[R_e(\rho/p_\infty)^{1/2}]$), two situations can be recognized: (1) the interval ΔT_y is longer than the bubble life, which will be symbolically denoted as $\Delta T_y \to \infty$, and (2) the length of the interval ΔT_y does not exceed the length of the bubble life, which will be denoted as $\Delta T_y < \infty$. According to the value of Δp^* three situations can be distinguished: (a) increased pressure, i.e., $\Delta p^* > 0$, (b) decreased pressure, i.e., $-1 < \Delta p^* < 0$, and (c) tension, i.e., $\Delta p^* \leqslant -1$. All possible combinations of ΔT_y and Δp^* can be considered briefly, as follows.

(1a) $\Delta T_y \to \infty$, $\Delta p^* > 0$. In this case equation (15) can be rearranged to the form of equation (5) by replacing variables Y with Z and P_m^* with $(1+\Delta p^*)^{-1}$. The period of bubble oscillations, T_o, is decreased by a factor $(1+\Delta p^*)^{-1/2}$. This technique was used, for example, by Smulders and van Leeuwen [17], who worked with relatively long pressure pulses.

(1b) $\Delta T_y \to \infty$, $-1 < \Delta p^* < 0$. Now equation (15) can be rearranged into the form of equation (10) by replacing variables Y with W and P_M^* with $(1+\Delta p^*)^{-1}$. The period of bubble oscillations, T_o, is increased by a factor $(1+\Delta p^*)^{-1/2}$.

(1c) $\Delta T_y \to \infty$, $\Delta p^* \leqslant -1$. Under these conditions equation (15) gives an unlimited growth; i.e., as $t_y \to \infty$, then $Y \to \infty$.

(2a) $\Delta T_y < \infty$, $\Delta p^* > 0$. For $0 \leqslant t_y \leqslant \Delta T_y$ the bubble behaves as in (1a) and one says that the bubble is driven during the interval ΔT_y. For $t_y > \Delta T_y$ the bubble performs oscillations in the equilibrium system and one can say that the bubble is released. An example of the computed bubble wall time history is given in Figure 3. Excitation by short pressure pulses was used in the work described in references [18–20].

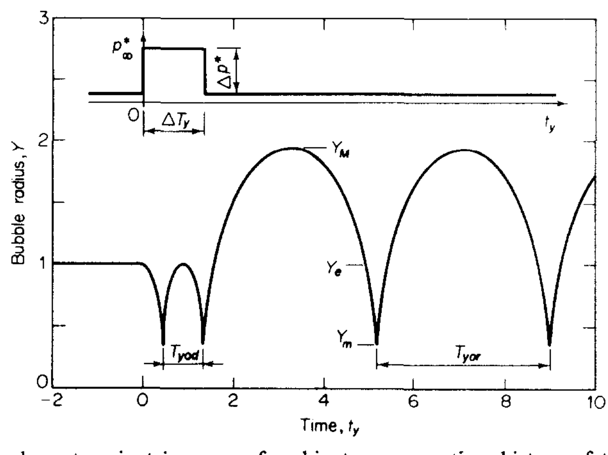

Figure 3. Excitation by a transient increase of ambient pressure: time history of the wall motion in the equilibrium system. $\Delta p^* = 5$, $\Delta T_y = 1{\cdot}33$, $\gamma = 4/3$. T_{yod}, T_{yor}—times of oscillation in the driving phase, and when the bubble is released. Y_M, Y_e, Y_m—maximum, equilibrium, and minimum bubble radii when the bubble is released.

(2b) $\Delta T_y < \infty$, $-1 < \Delta p^* < 0$. For $0 \leqslant t_y \leqslant \Delta T_y$ the bubble is driven and behaves as in (1b); for $t_y > \Delta T_y$ the bubble is released and performs oscillations in the equilibrium system. An example of the computed bubble wall time history is given in Figure 4.

(2c) $\Delta T_y < \infty$, $\Delta p^* \leqslant -1$. For $0 \leqslant t_y \leqslant \Delta T_y$ the bubble is driven and continuously grows; for $t_y > \Delta T_y$ the bubble is released and oscillates in the equilibrium system. An example

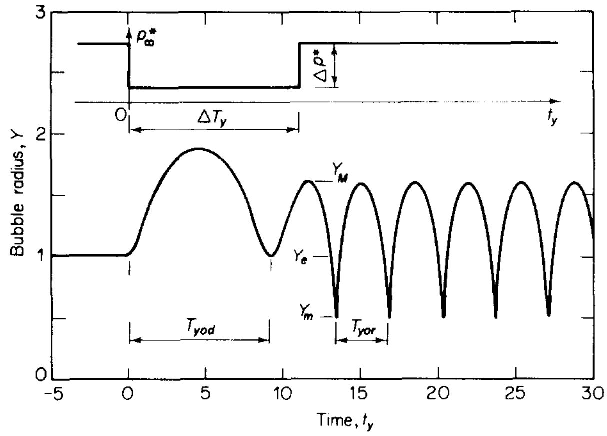

Figure 4. Excitation by a transient decrease of ambient pressure: time history of the wall motion in the equilibrium system. $\Delta p^* = -0{\cdot}75$, $\Delta T_y = 11$, $\gamma = 4/3$. T_{yod}, T_{yor}—times of oscillation in the driving phase, and when the bubble is released. Y_M, Y_e, Y_m—maximum, equilibrium, and minimum radii when the bubble is released.

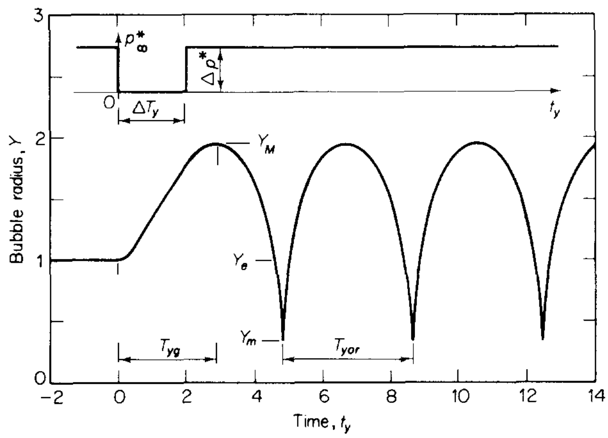

Figure 5. Excitation by a transient tension pressure: time history of the bubble wall motion in the equilibrium system. $\Delta p^* = 1$, $\Delta T_y = 2$, $\gamma = 4/3$. T_{yg}—time of bubble growth. T_{yor}—time of oscillation when the bubble is released. Y_M, Y_e, Y_m—maximum, equilibrium, and minimum radii when the bubble is released.

of the computed bubble wall time history is given in Figure 5. This kind of excitation may occur when a tension wave (e.g., a shock wave reflected at a free surface) travels through the liquid containing gas bubbles.

3. AMPLITUDE OF BUBBLE OSCILLATIONS

The intensity with which the bubble oscillates is determined by the values of the parameters P_m^*, P_M^*, or Δp^* and ΔT_y occurring in the equations of motion (5), (10) and (15), respectively. Another possibility for determining the intensity of oscillations is to make use of the initial conditions. This is used, for example, in the case of linear bubble oscillations [8].

To be able to compare different excitation techniques, a non-linear amplitude, A (here referred to simply as the "amplitude"), defined as

$$A = R_M / R_e \tag{16}$$

will be used. Here R_M and R_e are the maximum and equilibrium radii, respectively. The same measure of oscillation intensity was used, for example, by Lauterborn [21] and Flynn [22]. One can find the mutual relations between the non-linear amplitude A and the natural intensity parameters in different excitation systems, as follows.

3.1. COMPRESSION SYSTEM

As follows from equation (5) the natural intensity measure in the compression system is the minimum pressure P_m^*. The relation between P_m^* and A has the form

$$P_m^* = Z_e^{3\gamma} = A^{-3\gamma}. \tag{17}$$

Here $Z_e = R_e/R_M$.

Calculated variations of the amplitude A and of the compression time T_{zc} with the pressures P_m^* and Δp^* are given in Figure 6.

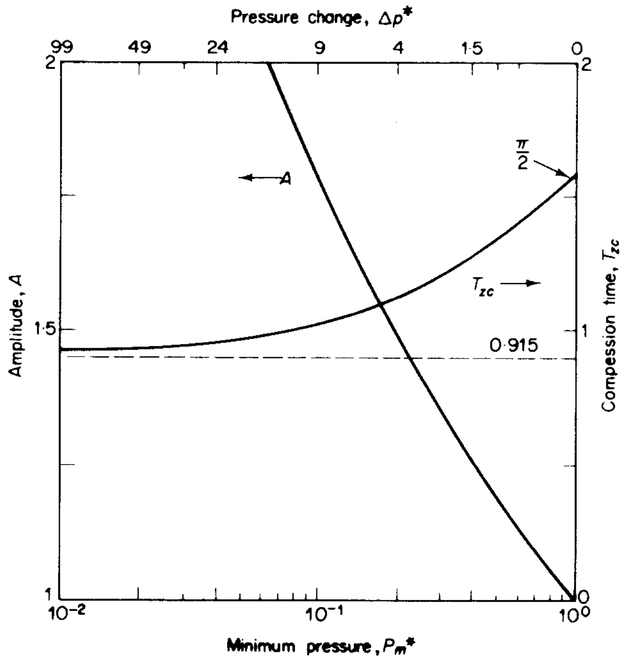

Figure 6. Compression system of bubble excitation: variation of the amplitude of bubble oscillation, A, and of the compression time, T_{zc}, with the minimum gas pressure, P_m^*, or the ambient pressure change Δp^*. $\gamma = 4/3$.

3.2. EXPANSION SYSTEM

The natural measure of oscillation intensity in this system is P_M^*. This maximum pressure is related to the amplitude A through the equation

$$P_M^* = W_e^{3\gamma} = (W_M/A)^{3\gamma}. \tag{18}$$

Here $W_e = R_e/R_m$ and $W_M = R_M/R_m$. For a given value of P_M^* the maximum bubble radius W_M can be determined by integrating equation (10). Computed variations of the amplitude, A, and of the expansion time, T_{we}, with the pressures P_M^* and Δp^* are given in Figure 7.

Figures 6 and 7 show that in the case of linear oscillations ($P_m^* \rightarrow 1$, $P_M^* \rightarrow 1$) the compression and expansion times are equal to $\pi/2$[8]. On the other hand, when the

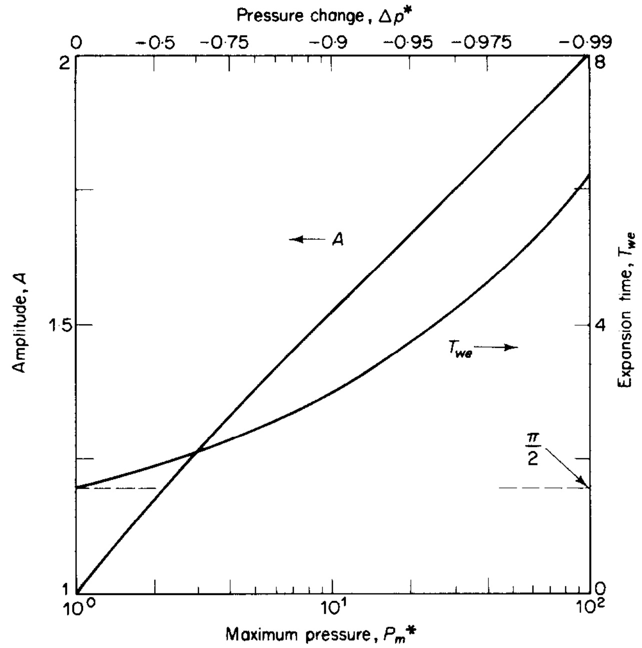

Figure 7. Expansion system of bubble excitation: variation of the amplitude of bubble oscillation, A, and of the expansion time, T_{we}, with the maximum gas pressure, P_M^*, or the ambient pressure change Δp^*. $\gamma = 4/3$.

minimum pressure P_m^* is decreased, the compression time quickly approaches the limiting value 0·915 valid for an empty bubble [7].

3.3. EQUILIBRIUM SYSTEM

As long as the bubble is driven the results of the preceding sections 3.1 and 3.2 can be used (with the exception of the case when $\Delta p^* \leq -1$). One needs therefore to consider the behaviour of the bubble only after it has been released. Hence, only the case $\Delta T_y < \infty$ needs to be studied.

For $t_y > \Delta T_y$ the amplitude A directly equals the maximum radius Y_M. As there is no analytical relation among Δp^*, ΔT_y, and A the mutual dependence of these quantities can be determined only by numerical integration of equation (15). Again, attention needs to be paid to the different ranges of Δp^* separately.

(2a) $\Delta T_y < \infty$, $\Delta p^* > 0$. Due to a periodicity in the bubble motion the dependence of A on ΔT_y is also a periodic function. It is therefore sufficient to examine only an interval $0 \leq \Delta T_y \leq T_{yod}$, where T_{yod} is a period of oscillations when the bubble is driven . For a particular pressure Δp^*, the period T_{yod} can be determined from Figure 6. The calculated dependences of A on ΔT_y for three different values of Δp^* are displayed in Figure 8. Figure 8 shows that if the pressure change lasts exactly $\Delta T_y = (k+\frac{1}{2})T_{yod}$, $k = 0, 1, 2, \ldots$, the bubble is excited to oscillate with a maximum amplitude. On the other hand, if the driving interval lasts exactly $\Delta T_y = kT_{yod}$, $k = 1, 2, \ldots$, the bubble, when released, ceases to oscillate completely. Thus excitation by this technique is very sensitive to the length of the interval ΔT_y.

(2b) $\Delta T_y < \infty$, $-1 < \Delta p^* < 0$. In this case also the motion of the bubble during the driving phase is periodic and it is therefore sufficient to consider the dependence of A on ΔT_y only for $0 \leq \Delta T_y \leq T_{yod}$. Calculated graphs are shown for three values of Δp^* in Figure 9. Again, the bubble is excited to oscillate with a maximum amplitude, A, if $\Delta T_y = (k+\frac{1}{2})T_{yod}$, $k = 0, 1, 2, \ldots$, and it ceases to oscillate completely if $\Delta T_y = k\ T_{yod}$, $k = 1,$

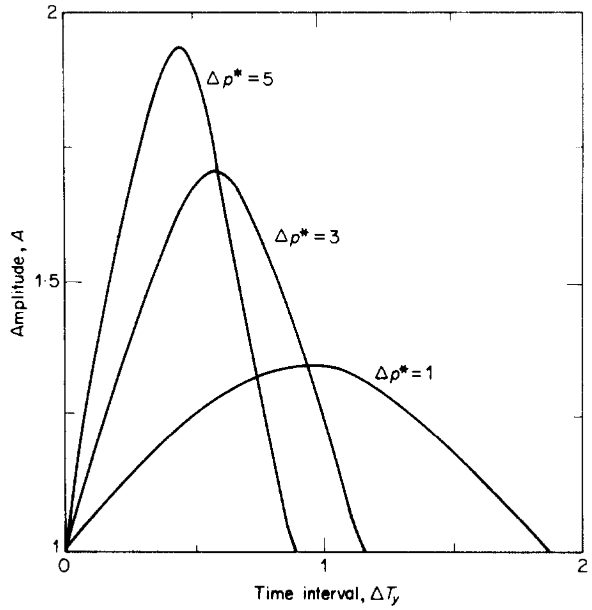

Figure 8. Excitation of a bubble by transient increase of ambient pressure: variation of the amplitude of oscillation when the bubble is released, A, with the length of the interval ΔT_y and the pressure change Δp^*. $\gamma = 4/3$.

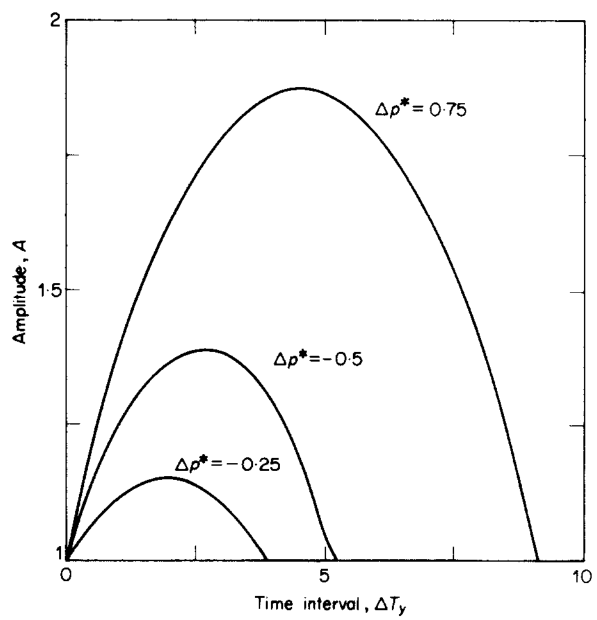

Figure 9. Excitation of a bubble by transient decrease of ambient pressure: variation of the amplitude of oscillation when the bubble is released, A, with the length of the interval ΔT_y and the pressure change Δp^*. $\gamma = 4/3$.

2, ... Therefore, excitation by this technique is also highly sensitive to the length of the interval ΔT_y.

(2c)$\Delta T_y < \infty$, $\Delta p^* \leq -1$. Now the bubble steadily grows during the driving phase. Therefore, both A and the growth time, T_{yg}, steadily increase with ΔT_y (see Figures 10 and 11).

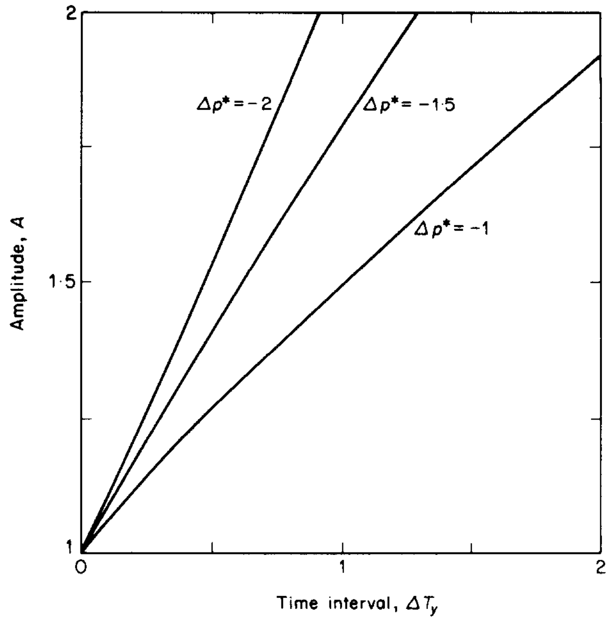

Figure 10. Excitation of a bubble by transient tension pressure: variation of the amplitude, A, with the length of the interval ΔT_y and the tension pressure Δp^*, $\gamma = 4/3$.

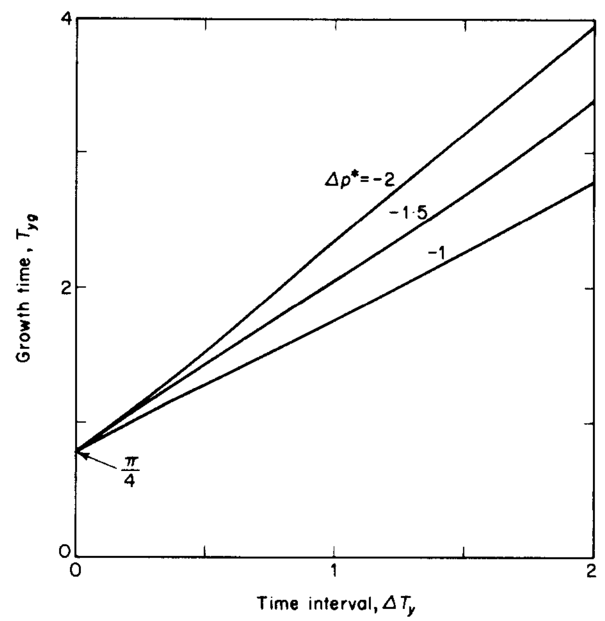

Figure 11. Excitation of a bubble by transient tension pressure: variation of the growth time, T_{yg}, with the length of the interval ΔT_y and the tension pressure Δp^*. $\gamma = 4/3$.

4. DISCUSSION

When comparing equations (1)–(5) with equations (6)–(10) it can be seen that the Z and W systems are mutually symmetric: i.e., under the transformations $Z \rightleftarrows W$, $m \rightleftarrows M$, and also in the case of the quantities denoted by the apostrophe $+ \rightleftarrows -$, the other system of equations will be obtained. Should the amplitude, A, defined by equation (16), be denoted as A_z and a quantity A_w be defined as $A_w = R_m/R_e$, then equations (17) and (18) also achieve a mutually symmetric structure. This symmetry is not only formal. For example, in Rayleigh's model it is sufficient to solve the equation of motion in only one of the two systems. The respective quantities in the other system can then be determined by simple algebraic operations. A certain kind of symmetry also holds with respect to excitation by increased and decreased ambient pressure in the Y system.

In the limit of small amplitudes ($A \to 1$), the wall motion is also symmetric around the equilibrium radius R_e and for a given linear amplitude $A_x = (R_M - R_e)/R_e = A - 1$ then $P_M^* = 1 + 3\gamma A_x$ and $P_m^* = 1 - 3\gamma A_x$ [8]. However, due to the inherent non-symmetry in the bubble wall motion (a divergence during the expansion phase and a convergence during the compression phase), when the amplitude of oscillations is increased the maximum pressure P_M^* grows much faster and the minimum pressure P_m^* much more slowly than predicted by the formulae for the linear case (cf. also Figures 6 and 7). This means that smaller absolute deviations from p_∞ are necessary in the Z system than in the W system to excite bubbles to the same amplitudes. The same is true when comparing excitations by increased or decreased ambient pressure in the Y system. In this case, when the length of the interval ΔT_y equals approximately one half of the oscillation period T_{yod}, the amplitude of oscillations after the bubble is released is even larger than that in the driving phase. Finally, the excitation by tension seems to be the most effective one: the amplitude increases without restrictions with an increase in the tension pressure Δp^* as well as in the length of the interval ΔT_y. Thus, excitation for larger amplitudes can be accomplished by this technique relatively easily.

It was shown that each excitation system has its own inherent measure of oscillation intensity. However, there is an obvious need (e.g., for the purpose of comparison) to use one universal measure. Such a universal measure can be chosen in several ways: e.g., it can be one of the natural measures P_m^* or P_M^*, or some significant wall position such as Z_m, W_e, etc. Here the measure $A = Y_M = 1/Z_e$ was chosen because it is easy to use and it allows a quick interpretation (and comparison) of results in cases of both free and forced, and linear and non-linear, oscillations.

From the point of view of evaluating experimental data the Z system proves to be advantageous and hence its widespread use is fully justified. However, when considering the proper initial conditions in theoretical computations (e.g., the initial temperature, or the upward velocity of the bubble) the particular way the bubble arrived at Z_M must be considered. Unfortunately, there are not many studies on bubble dynamics in which a particular excitation technique has been specified. From the few works where this has been done one can mention at least those of references [22] and [23], in which the bubble growth due to a tension pulse was studied.

The physical differences among the three excitation techniques can be best seen when considering the temperatures the gas inside the bubble attains during oscillations. Suppose the initial gas temperature is equal to the liquid temperature at infinity, Θ_∞ (usually the room temperature). Then in the case of excitation by decreasing bubble energy the gas temperature will always be equal to or higher than the room temperature and it will oscillate between the minimum temperature $\Theta_m = \Theta_\infty$, when $Z = Z_M$, and a maximum temperature $\Theta_M = \Theta_\infty Z_m^{-3(\gamma-1)}$, when $Z = Z_m$ (Figure 1).

On the other hand, in the case of excitation by increasing bubble energy the gas temperature will always be equal to or lower than the room temperature. It will oscillate between the maximum temperature $\Theta_M = \Theta_\infty$, when $W = W_m$, and a minimum temperature $\Theta_m = \Theta_\infty W_M^{-3(\gamma-1)}$, when $W = W_M$ (see Figure 2).

Finally, in the case of excitation by a transient change of the ambient pressure, when the bubble is released the gas temperature will oscillate around the room temperature. It will attain a minimum value $\Theta_m = \Theta_\infty Y_M^{-3(\gamma-1)}$, when $Y = Y_M$, and a maximum value $\Theta_M = \Theta_\infty Y_m^{-3(\gamma-1)}$, when $Y = Y_m$ (see Figures 3-5).

In order to illustrate these formulae one can consider a simple example. Suppose that $\gamma = 4/3$, $\Theta_\infty = 300$ K, and $R_M/R_m = 10$. Then in the first case one has $Z_m = \cdot 1$, $\Theta_m = 300$ K, and $\Theta_M = 3000$ K. In the second case one has $W_M = 10$, $\Theta_M = 300$ K, and $\Theta_m = 30$ K. Finally, in the third case, if one further assumes that $Y_M = 2$ and $Y_m = 0{\cdot}2$ (which gives the required ratio $R_M/R_m = 10$), one obtains $\Theta_m = 150$ K and $\Theta_M = 1500$ K.

Hence, whereas in experiments with evacuated glass spheres [9, 10] it has been possible to observe light flashes due to the gas ionization (sonoluminescence), these light flashes should not be seen (providing the thermal theory of sonoluminescence [1] is correct) in experiments with pressurized glass spheres [14], even if the ratio R_M/R_m is maintained to be the same in both experiments.

Closely related to the gas temperature is the speed of sound in the gas. In order to examine a possibility for development of converging spherical shocks in the bubble interior, the sound speed has been studied in great detail [24]. Again, distinct differences among the various excitation techniques were revealed.

Another difference among the three techniques concerns the maximum attainable amplitudes of the bubble oscillations. Some difference between the excitation methods can already be seen in Figures 6 and 7, but this difference becomes much more distinct for higher and lower initial pressure P_M^* and P_m^*, respectively, than for those considered here. For those higher and lower initial pressures, however, computations can be performed only with models assuming compressible liquids and therefore the author proposes to discuss this question elsewhere.

Though models based on the assumption of liquid compressibility would give more correct results [25], for this study Rayleigh's model has been deliberately chosen, and hence a non-compressible liquid has been assumed. The advantage of such an approach is apparent in the much simpler derivations of the equations of motion. For example, in the case of excitation by pressure change the derivation of the equation of motion represents, due to the scattering of the incident driving wave on the spherical bubble, a very complicated task. Another advantage appears to be much more transparent relations among Δp^*, ΔTy and A in the case of excitation by a pressure pulse. Finally, much simpler notation can be used in connection with non-dissipative models.

An obvious disadvantage of such an approach is the absence of radiation damping in Rayleigh's model. To minimize the consequent error only bubbles oscillating with moderate amplitudes ($A \leq 2$) have been studied, deliberately [25].

The reasons why the derivation of the otherwise well-known equations of motion have been included in this study are briefly as follows. First it was desirable to show the close analogy between the mathematical formulation and the experimental techniques. Second, the designation of the particular excitation method can be best traced from the mathematical description of the processes studied (e.g., at the time $t = 0$). Last but not least the author believes that completeness and clarity of the presentation are gained thereby.

It has been assumed that no evaporation or condensation takes place during the bubble oscillations. Evidently, as long as the ambient pressure is above the liquid vapour pressure, this assumption will not be violated. It is the author's opinion that even if evaporation or condensation occurs the wall motion of the gas bubble (oscillating with a moderate amplitude) will be little influenced. However, for larger amplitudes the speed of condensation may be a limiting factor.

There is a very important class of so-called vapour bubbles which are supposed to contain only the vapour of the surrounding liquid. The importance of the vapour bubbles follows from the role they play in cavitation and boiling [26]. Excitation of these bubbles has not been considered here, though the author believes that a classification scheme, in many respects similar to that presented here, may also be devised for them.

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