Time-Frequency Analysis of Cavitation Noise

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A model of cavitation noise based on random group pulse processes is presented. The model is used to simulate cavitation noise on a computer. The generated signal is analysed using both a stationary and cyclostationary approach. In the first case autospetral density is computed. In the second case a time-frequency representation is obtained. The time-frequency representation makes it possible to obtain cavitation statistics for different times within the driving pressure field period.

INTRODUCTION

When a liquid is irradiated by a sufficiently intensive periodic sound wave, cavitation bubbles oscillating in a very complicated non-linear way are generated. The oscillations of bubbles are accompanied by radiation of pressure waves into surrounding liquid. Due to random character of bubble occurrence the pressure waves are also random and hence are called cavitation noise. Cavitation noise has been studied for a long time (see, e.g., references [1-3]). However, not all aspects of this complex phenomenon have been fully clarified yet.

In this paper a mathematical model of cavitation noise will be given. This model is based on the theory of random group pulse processes [4]. Using the model, cavitation noise is simulated on a computer. The generated signal is then analysed using both a classical autospectral density and a time-frequency statistical characteristics.

CAVITATION NOISE MODEL

The bubbles radiate significant acoustic energy only for a short time interval when in the vicinity of their minimum volume. Hence, to a first approximation, the radiated waves may be considered to be a sequence of pressure pulses occurring in random groups with mean time interval between them equal to the period of the driving field *T*. The pressure pulses can be approximated by a double-sided exponential function [5]

$$p_1(t) = P\exp\left(-\left|t\right|/\theta\right) \quad . \qquad (1)$$

The peak pressure P and time constant θ of the pressure pulse can be estimated from the scaling functions published in reference [6]. These scaling functions were computed using a Gilmore's model. Assuming the amplitude of bubble oscillations A=3.5 and the adiabatic exponent for the bubble content (vapour) γ=1.25 [5], the corresponding non-dimensional peak acoustic pressure is P_z=200, and non-dimensional pulse effective width ϑ_z=10^{-3}. Assuming further that the maximum bubble radius is R_M=1 mm and that cavitation noise is measured at a distance r=0.1 m, the dimensional peak acoustic pressure is P=3x10^5 Pa and pulse effective width ϑ=0.1 μs. The calculated values correspond to water (ρ_∞=10^3 kg.m^{-3}) under atmospheric pressure p_∞=10^5 Pa. For the pressure pulse given by equation (1) the effective pulse width ϑ is equal to the time constant θ.

The mathematical model of cavitation noise *p(t)* can be then written in the form [4]

$$p(t) = \sum_{k=-\infty}^{\infty}\sum_{n=1}^{N_k} P_{kn}\exp(-\left|t-kT-\varphi_{kn}\right|/\theta_{kn}) \quad . \qquad (2)$$

Here N_k is a random number of pressure pulses in the kth group, P_{kn} is a random peak acoustic pressure of the nth pulse in the kth group, φ_{kn} is a random distance of the nth pulse in the kth group from a reference point of the kth group, and θ_{kn} is a random time constant of the nth pulse in the kth group [4].

CAVITATION NOISE SIMULATION

Cavitation noise *p(t)* given by eq. (2) has been simulated on a computer. All random variables (i.e., N, P, φ, and θ) have been assumed to be mutually independent. The random variables N have been assumed to be Poisson distributed with a mean value μ_N=20. The random variables P, φ, and θ have been assumed to be normally distributed with mean values and standard deviations, respectively, μ_P=3x10^5 Pa, σ_P=7.5x10^4 Pa, μ_φ=0 s, σ_φ=1 μs, μ_θ=0.1 μs, σ_θ=0.025 μs. The driving pressure field frequency was f_o=20 kHz (T=50 μs) and the sampling frequency was f_s=82 MHz. An example of one cavitation pulse group is given in Fig. 1.

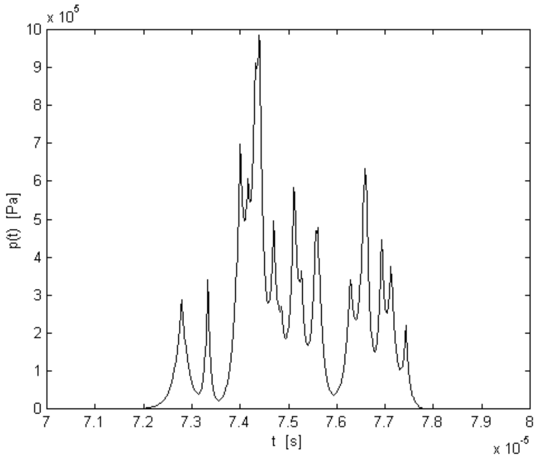

FIGURE 1. A selected part from a simulated cavitation noise signal *p(t)* showing one group of random pressure pulses.

CAVITATION NOISE ANALYSIS

Cavitation noise signal generated on a computer has been analysed using both classical autospectral density and a new approach based on time-frequency analysis of cyclostationary signals.

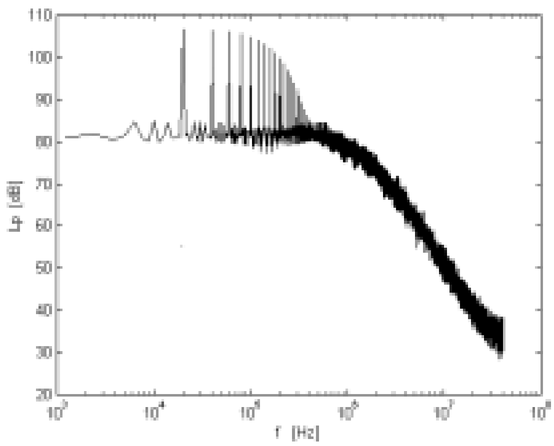

FIGURE 2. Computed autospectral density of simulated cavitation noise.

The computed autospectral density $P_{xx}(f)$ is shown in Fig. 2. It has two parts: a discrete autospectrum consisting of the basic and higher harmonic components, and a continuous autospectrum. The overall agreement between the simulated and measured autospectra [1, 2] is good, with the exception of subharmonic and ultraharmonic components [3], which are not present in the simulated signal.

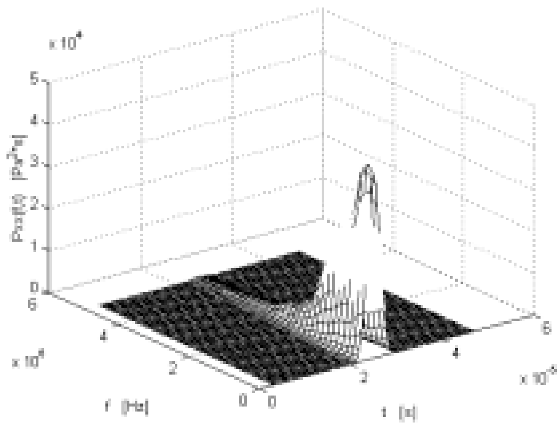

FIGURE 3. Computed gated autospectral density of simulated cavitation noise.

The random part of the simulated cavitation noise has been then analysed using a gated autospectral density $P_{xx}(f,t)$ [7]. The computed characteristics is shown in Fig. 3. As can be seen the gated autospectrum makes it possible to reveal the instants when the pressure pulses are radiated. This information could not be determined from the classical autospectral density.

CONCLUSIONS

Cavitation noise is a very complex phenomenon not fully understood yet. The method of time-frequency analysis could help to clarify some questions. The simulation studies may be useful in designing new experiments and in evaluating experimental data.

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