

DŮM TECHNIKY ČSVTS PRAHA



TECHNICKÁ DIAGNOSTIKA '82

SBORNÍK REFERÁTŮ

1982

STATISTICAL ANALYSIS OF
DIAGNOSTIC SIGNALS WITH
RANDOM PERIODIC STRUCTURE

Karel Vokurka

Abstract: Diagnostic signals with random periodic structure are emitted by a number of important machines. These signals may be treated as being either stationary or non-stationary. The first approach (stationary) is currently used in applications. However, it is shown that the second approach (nonstationary) can be more convenient because of greater information content of the corresponding statistical characteristics.

Key words: ACOUSTICAL DIAGNOSTICS OF MACHINERY, SIGNALS WITH RANDOM PERIODIC STRUCTURE, PERIODICALLY NON-STATIONARY PROCESSES

1. Introduction

Diagnostic signals may be roughly divided into two groups. The first group comprises simple diagnostic signals such as output voltages from thermocouples. The second group comprises more complex diagnostic signals that occur, for example, at acoustical diagnostics of machinery. To find the diagnostic symptoms in such signals, rather complex apparatus are necessary (e.g. correlators, spectrum analyzers etc.). Even with those apparatus it may sometimes be impossible to identify the symptoms. The reason for it lies in insufficient information content of the measured statistical characteristics. In this paper we want to draw attention to the characteristics of the signals with periodic structure, information content of which may be increased by suitable treatment.

2. The random process with periodic structure

With respect to the behaviour in time the random processes are distinguished as being either stationary or nonstationary [1]. There is a special subclass of the random processes that comprises the processes with periodic structure (some authors also call them the periodically nonstationary processes - e.g. [2]). These processes are principally nonstationary, however, when suitably treated they also can be considered as stationary. Because they are often encountered in applications they are of great importance.

Let us consider an ensemble of random process with periodic structure, $x(t, \varphi)$, where the phase φ is supposed to be constant (Fig. 1a). It appears on first inspection that the statistical characteristics of this process are time dependent and thus the process $x(t, \varphi)$ is nonstationary. However, if the phase φ is considered to be random and uniformly distributed in the primary interval $(0, T_0)$, the process $x(t, \varphi)$ will be stationary [1] (Fig. 1b).

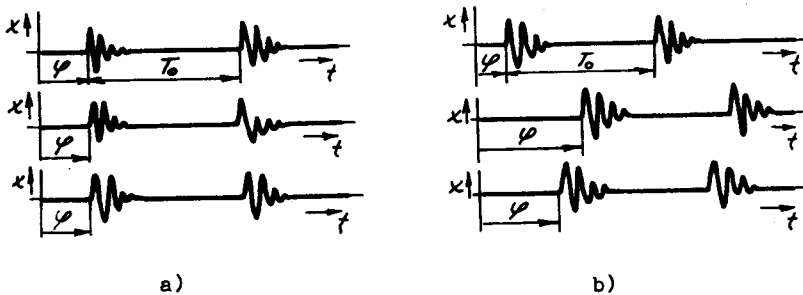


Fig. 1

The basic difference between the statistical characteristics obtained in these two different approaches is that the characteristics in the second (stationary) case represent time averages of the characteristics in the first

(nonstationary) case. Let us note that all conventional analyzers use the stationary approach. However, as any averaging procedure is accompanied by irreversible loss of information, the information content of the characteristics in the case of stationary approach is necessarily smaller. Let us illustrate this by an example of the autocorrelation function and the power spectrum of the process $x(t, \varphi)$. It is evident that similar results can be also obtained with other statistical characteristics, such as the variance or the first probability density. To simplify the notation the symbol φ will be dropped.

3. The autocorrelation function and the power spectrum

The process $x(t)$ may be broken up into two basic components, namely

$$x(t) = s(t) + n(t), \quad (1)$$

where $s(t) = s(t+T_0) = \langle x(t) \rangle$ is the ensemble average of the process $x(t)$ and $\langle n(t) \rangle = 0$. The autocorrelation function of the process $x(t)$ can be found to be

$$R_x(t_1, \tau) = R_s(t_1, \tau) + R_n(t_1, \tau). \quad (2)$$

It has two terms. The first term, $R_s(t_1, \tau)$, is the autocorrelation function of the component $s(t)$ and it is a periodic function of both t_1 and τ . The second term, $R_n(t_1, \tau)$, is the autocorrelation function of the random component $n(t)$ and it may (but need not) be a periodic function of t_1 .

The power spectrum of the process $x(t)$ can be determined by taking the Fourier transform of the autocorrelation function (2). Hence

$$W_x(t_1, \omega) = W_s(t_1, \omega) + W_n(t_1, \omega). \quad (3)$$

Here W_x , W_s and W_n are the respective Fourier transforms of R_x , R_s and R_n . The component W_s is a periodic function of

t_1 , the component W_n may (but need not) be a periodic function of t_1 (Fig. 2).

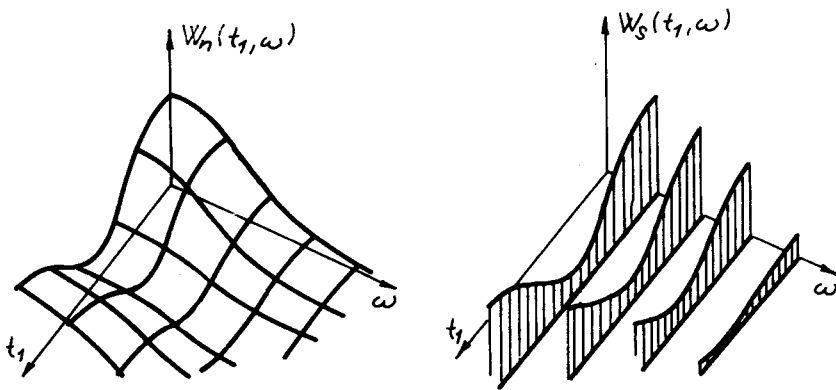


Fig. 2

Now we can determine the time averages of (2) and (3). We obtain

$$\frac{1}{T_0} \int_0^{T_0} R_x(t_1, \tau) dt_1 = R_x(\tau) = R_s(\tau) + R_n(\tau) \quad (4)$$

and

$$\frac{1}{T_0} \int_0^{T_0} W_x(t_1, \omega) dt_1 = W_x(\omega) = W_s(\omega) + W_n(\omega). \quad (5)$$

An example of the power spectra $W_s(\omega)$ and $W_n(\omega)$ is given in Fig. 3.

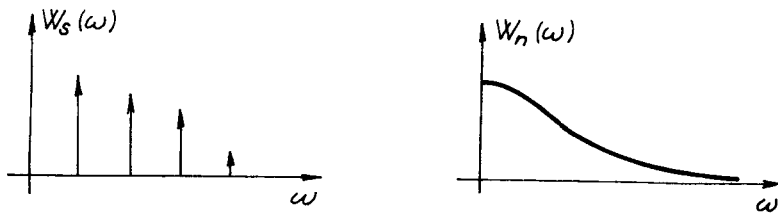


Fig. 3

In the process of averaging a certain information about the process $x(t)$ was lost. This information concerns, for example, the phases of the harmonic components of $s(t)$ and the periodicity of R_n or W_n . The form of the function $s(t)$ can be determined by the crosscorrelation of the process $x(t)$ with periodic impulses. The autocorrelation function $R_x(t_1, \tau)$ or the power spectrum $W_x(t_1, \omega)$ may be determined using the periodic time window $w(t)$ [3], [4] (Fig. 4). The characteristics are periodically detected in small intervals ΔT , centered at different positions t_1 , and are averaged over the statistical ensemble, made up of sequence of successive intervals ΔT .

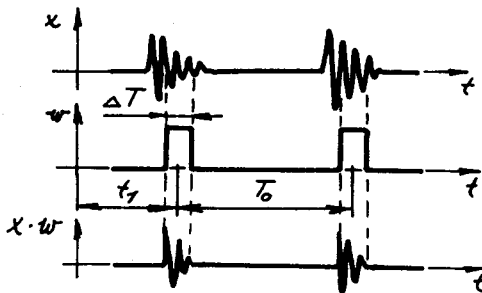


Fig. 4

Selection of the actual value of ΔT represents a certain compromise. To obtain sufficient resolution in t_1 , ΔT should be chosen as small as possible. However, if ΔT is decreased, the frequency resolution Δf is also decreased. For example, let us suppose that we want to determine the power spectrum $W_x(t_1, \omega)$ of the diagnostic signal from a diesel engine running at 1500 rpm. If the time window ΔT is chosen to be 10 % of T_0 , that is $\Delta T=4$ ms, then the frequency resolution is $\Delta f=1/\Delta T=250$ Hz (for the rectangular time window). For 1 % resolution in t_1 , $\Delta T=0,4$ ms and $\Delta f=2,5$ kHz.

References

- [1] Middleton, D.: An Introduction to Statistical Communication Theory. McGraw - Hill, New York 1960.
- [2] Gudzenko, L. I.: O periodičeski nestacionarnych processach. Radiotekhnika i elektronika, 4, 1959, 6, 1062 - 4.
- [3] Hanke, V.: Některé způsoby vyhodnocování emitovaného ultrazvukového stochastického signálu. Conf.: Bezdemonstrážní diagnostika v průmyslu, ČVTS, Praha 1975, 31-36.
- [4] Randall, R. B.: Diagnostics of slow-speed reciprocating machines. Conf.: Technical diagnostics '79, Karlovy Vary 1979, 159-68.