

Acoustical approach to analysis of energy conversions in an oscillating bubble

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Abstract Acoustic radiation of freely oscillating bubbles has been studied experimentally. The oscillating bubbles have been generated by underwater spark discharges. Recorded pressure waves made it possible to determine the acoustic energy radiated by the oscillating bubbles and potential energy of the bubbles when attaining the first and second maximum volumes. Comparison of the potential and acoustic energies revealed that about 70 % of the potential energy of the bubble available at the first maximum volume has been converted into an unidentified energy. A possible candidate for the yet unknown energy conversion mechanism seems to be vapor condensation and evaporation, converging shock wave in the bubble interior and turbulence in the liquid when the bubble compression and expansion are not symmetric. The measured energies are compared with energies computed using a suitable theoretical model. These comparisons also show that the present theoretical models do not encompass all energy conversions.

1 INTRODUCTION

Free oscillations of bubbles are accompanied by a number of energy conversions. In the case of gas scaling bubbles the present theoretical models assume potential energy of the bubble-liquid system, internal energy of the gas in the bubble interior, kinetic energy of the flowing surrounding liquid and acoustic energy radiated by the bubble [1]. In the presentation results of experimental investigation of acoustic radiation from freely oscillating bubbles are given. It will be shown, among others, that there are energy conversions in an oscillating bubble, which are not explained at present time satisfactorily. Results presented here are an extension of the work published in reference [2].

2 EXPERIMENTAL SETUP

Freely oscillating bubbles have been generated by discharging a capacitor bank via a sparker submerged in water. The discharges took place in a laboratory water tank having dimensions 6 m x 4 m x 5.5 m. The experimental setup is schematically shown in Figure 1 and a detailed description of it can be found in reference [3].

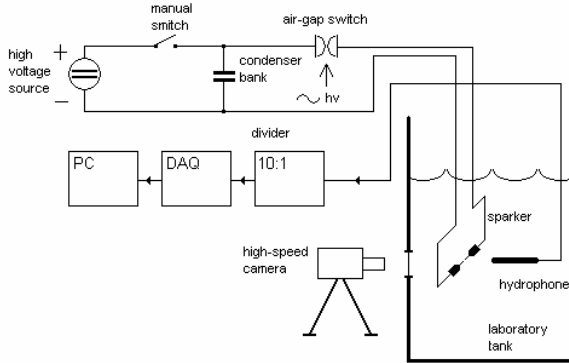


Figure 1. *Experimental setup used to study spark generated bubbles*

The pressure waves radiated by the bubbles were recorded with a broadband hydrophone (Reson type TC 4034) having a usable frequency range from 1 Hz to 470 kHz. The hydrophone was positioned at distances from the bubble center $r = 0.1$ m, 0.25 m, and 0.5 m. The hydrophone output was connected via a voltage divider to a data acquisition board (National Instruments PCI 6115, 12 bit A/D converter) having a sampling frequency of 10 MHz. The experimental bubbles could be recorded using a 16-mm film rotating prism high-speed camera.

3 PRESSURE RECORDS

Using the experimental arrangement described above we obtained 448 pressure records and 10 film records. An example of a typical pressure record is given in Figure 2.

The recorded pressure consists of several pulses: of an initial pressure pulse $p_0(t)$, which is radiated during the spark discharge, and of the first, second, and third bubble pulses $p_1(t)$, $p_2(t)$, and $p_3(t)$, respectively. These bubble pulses are radiated when the pulsating bubble is compressed to its minimum volumes.

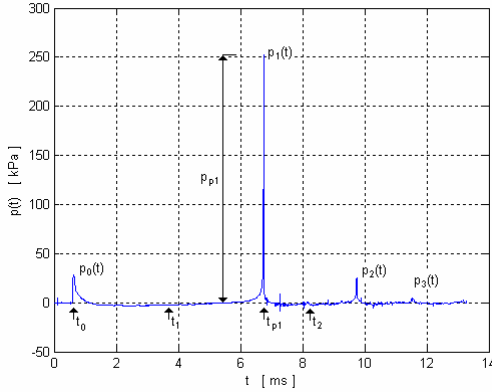


Figure 2. *A pressure wave radiated by an oscillating spark generated bubble measured at $r = 0.1$ m*

In this record t_0 denotes the start of the wave and t_{p1} the time of occurrence of the first peak in the wave. The first time of the bubble oscillations is then $T_{o1} = t_{p1} - t_0$. The instants t_1 and t_2 correspond to times the bubble attains maximum volumes. Finally, p_{p1} is the peak pressure in the first bubble pulse.

A freely oscillating spherical bubble is basically described by its size and intensity of oscillations. A suitable measure of the bubble size is the first maximum radius R_{M1} . This quantity can be easily determined from a pressure record using a well known formula:

$$R_{M1} = \frac{T_{o1}}{1.84 \sqrt{\frac{\rho_\infty}{p_\infty}}} \quad . \quad (1)$$

Here ρ_∞ is liquid density and p_∞ is ambient pressure in the liquid at the place of the bubble. The second maximum radius R_{M2} can be computed similarly from T_{o2} .

A suitable measure of the bubble oscillation intensity is the non-dimensional peak pressure in the first bubble pulse p_{zp1} defined by equation

$$p_{zp1} = \frac{p_{p1}}{p_\infty} \frac{r}{R_{M1}} \quad . \quad (2)$$

Here r is the distance from the bubble center to the hydrophone.

From the pressure records we were able to determine for each bubble its size, as represented by the maximum bubble radius, R_{M1} , and intensity of oscillations, as represented by the non-dimensional peak pressure in the first bubble pulse, p_{zpl} . The bubble sizes R_{M1} and bubble oscillation intensities p_{zpl} determined in this way are summarized in the scatter plot (in the so called bubble map) given in Figure 3.

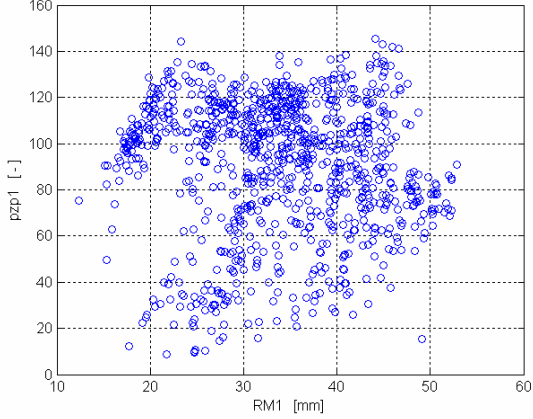


Figure 3. *Variation of the bubble oscillation intensity p_{zpl} with the bubble size R_{M1} (the bubble map) for experiments with spark generated bubbles*

It can be seen that the generated spark bubbles were relatively large (R_{M1} ranging from 12 mm to 53 mm) and oscillating with a broad range of intensities (p_{zpl} ranging from 9 to 145).

4 DISCUSSION

The analysis of the recorded pressure waves allowed us to obtain a number of interesting results. In this section we would like to discuss energy conversions in an oscillating bubble. Let us briefly mention the energies involved.

At a moment t_1 , when the bubble attains the first maximum volume, the bubble-liquid system has a potential energy E_{p1} and the gas in the bubble interior internal energy E_{i1} . In the following instants the potential energy E_{p1} is gradually converted into the inner energy of the compressed gas E_i , kinetic energy of the surrounding liquid E_k , acoustic energy ΔE_a carried away from the bubble by the radiated pressure wave, and at present time unidentified energies, which will be summarily denoted E_u . Thus the energy balance can be expressed as

$$E_{p1} + E_{i1} = E_p + E_i + E_k + \Delta E_a + E_u \quad . \quad (3)$$

In this equation the left hand side corresponds to time t_1 and the right hand side to any time $t > t_1$. In the following we shall limit ourselves to the instant $t = t_2$ which corresponds to the second maximum bubble volume. In this case we can compute E_{p1} , E_{p2} , and ΔE_a from the pressure records relatively easily (see, e.g. reference [3]). If we further assume that $E_{i2} \approx E_{i1}$ and $E_{k2} \approx E_{k1} \approx 0$, then we can compute the unaccounted for energy from the above equation and express it in a non-dimensional form

$$E_{zu} = \frac{E_{p1} - E_{p2} - \Delta E_a}{E_{p1}} 100 \quad [\%] \quad . \quad (4)$$

The unaccounted for energies calculated from the measured pressure records are given in Figure 4.

As can be seen, as much as 80 % of the potential energy available at the time t_1 has been converted into the unaccounted for energies at the time t_2 . And it is also interesting to note that a larger portion of the potential energy is converted into unaccounted for energies for less intensively oscillating bubbles than in the case of the more intensively oscillating bubbles.

Experimental evidence available at present time is insufficient to explain the nature of the unaccounted for energy (energies) satisfactorily. One can only speculate. Is it a condensation and evaporation energy? Is it energy dissipated in a converging wave in the bubble interior? Or is turbulence due to non-symmetric bubble motion responsible for some energy losses? And so on.

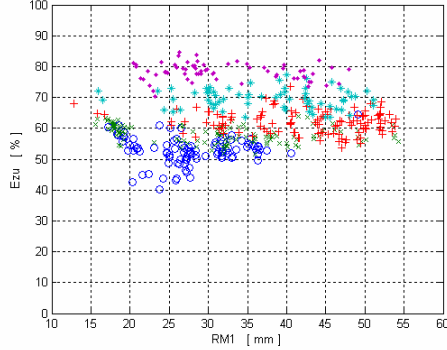


Figure 4. Variation of unaccounted for energy E_{zu} with bubble size R_{M1} . Experimental data:
 • $-p_{zpl} < 40$, * $-40 < p_{zpl} < 60$, + $-60 < p_{zpl} < 80$, x $-80 < p_{zpl} < 100$, o $-p_{zpl} > 100$

Using theoretical models of oscillating gas bubbles based on Gilmore's equation and Herring's modified equation (for definition of these models see, e.g. reference [4]) we have calculated radiated acoustic energies and damping factors for bubbles oscillating with different intensities. These theoretical results will be now briefly compared with experimental data in the following figures.

Figure 5 shows variation of the non-dimensional acoustic energy in the first bubble pulse with bubble oscillation intensity. The non-dimensional acoustic energy is defined by the following equation

$$\Delta E_{za} = \frac{\Delta E_a}{E_{p1}} 100 \quad [\%] \quad (5)$$

Note that the acoustic energies radiated by real bubbles are lower by about 10 % to 15 % than those computed in theoretical models. Acoustic radiation in Gilmore's model is lower than that in modified Herring's model, but still higher than in real bubbles.

In Figure 6 variation of the damping factor α_l with bubble oscillation intensity is given. The damping factor is defined by the following equation

$$\alpha_1 = \frac{R_{M2}}{R_{M1}} \quad (6)$$

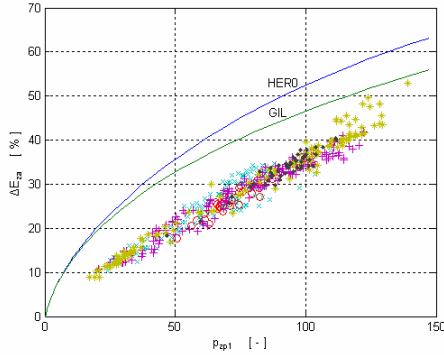


Figure 5. Variation of non-dimensional acoustic energy ΔE_{za} with p_{zp1} . Experimental data: $\bullet - R_{MI} < 20$, $* - 20 < R_{MI} < 30$, $+ - 30 < R_{MI} < 40$, $x - 40 < R_{MI} < 50$, $o - R_{MI} > 50$, all dimensions are in mm. Theoretical curves: GIL – Gilmore’s model, HERO – Herring’s modified model

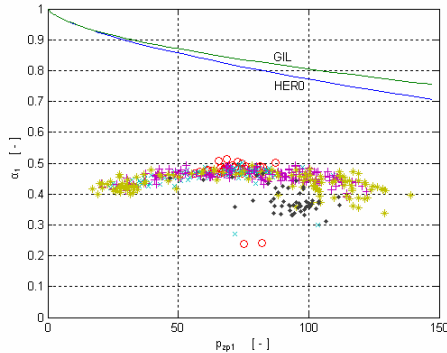


Figure 6. Variation of the damping factor α_1 with p_{zp1} . Experimental data: $\bullet - R_{MI} < 20$, $* - 20 < R_{MI} < 30$, $+ - 30 < R_{MI} < 40$, $x - 40 < R_{MI} < 50$, $o - R_{MI} > 50$, all dimensions are in mm. Theoretical curves: GIL – Gilmore’s model, HERO – Herring’s modified model

Now the modified Herring’s model gives better results than Gilmore’s model. This is thanks to higher acoustic radiation in the modified Herring’s model. However, even now the theoretical results are much higher than the experimental points.

5 CONCLUSIONS

From the analysis presented above and from some other facts, which for space limitations could not be mentioned here it seems highly probable that the origin of the unidentified energies should be sought in the bubble interior. The liquid and its compressibility seem to be described in both theoretical equations (Gilmore's and Herring's) well. However, the bubble interior represents a very complex, non-homogeneous, and non-equilibrium medium, which requires further both theoretical and experimental investigations.

It is strongly believed that the results obtained by studying the spark-generated bubbles have a more general validity. It can be expected that similar phenomena as discussed here will be also observed in laser-generated bubbles, and in cavitation bubbles generated in hydraulic machinery. But we also believe that to a certain degree these results are also valid for bubbles generated during acoustic cavitation, including the single-bubble cavitation so intensively studied during the past two decades.

ACKNOWLEDGMENTS

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