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Measurement and interpretation of instantaneous autospectrum of cyclostationary noise

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Abstract [762] Traditional spectral analysis is based on an assumption of analyzed noise stationarity and hence yields only time independent results. However, in the case of noise generated by cyclically working machinery it is possible to make use of the noise cyclic character and obtain instantaneous autospectrum showing spectral composition of the noise in any time instant. Information content of this statistical characteristic is higher when compared with traditional approach. This can be exploited in a number of applications including noise reduction, predictive maintenance, and quality control. In the paper some practical aspects of measurement and interpretation of instantaneous autospectrum of the cyclostationary noise are reported.

1 INTRODUCTION

A time-varying function *x(t)* is called periodic, if it fulfills the following relation [1]

$$x(t) = x(t \pm nT), \qquad n = 1,2,... \quad . \tag{1}$$

Here $T>0$ is a period.
The periodic function is an abstract mathematical notion. Though many reciprocating machinery generate noise and vibration whose time records may, to a certain degree, fulfill equation (1), due to many random effects present in a real life these records will always more or less depart from equation (1). The real reciprocating machinery simply does not work periodically but cyclically and hence generate noise and vibration, which are not periodic but cyclic.
Periodic functions are relatively simple to analyze as all what is needed to know for their complete description is either a time record of one period or knowledge of amplitudes and initial phases of harmonic components, into which the periodic function can be decomposed. This decomposition has been known for almost 200 years and belongs to standard undergraduate curricula.
In contrast to the periodic functions, a complete analysis of cyclic functions represents much tougher problem, which, in spite of great efforts, has not been solved satisfactorily yet. The interest in the complete analysis of the cyclic functions is due to widespread occurrence of the cyclic data in nature and technology, including the field of noise and vibration.

An interesting feature of the cyclic noise and vibration signals is that they may be treated either as stationary signals, whose statistical characteristics are time invariant, or nonstationary signals, whose statistical characteristics vary cyclically with time.
The stationary approach to the cyclic noise and vibration, which in theoretical works is based on the so called phase randomization and in experiments on continuous time averaging, has been successfully used for a long time and belongs to the standard signal analysis methods [1]. Unfortunately the stationary approach exploits only a part of information carried by the cyclic signals. Remaining information is lost during continuous time averaging.
In the last fifty years efforts have been reported in the literature to enhance the analysis of the cyclic signals by considering their nonstationarity. An extensive literature exists on this topic. Here at least a classical textbook by Gardner [2] should be mentioned. The nonstationary approach to the cyclic signal analysis has been studied by communication engineers first of all. However, in recent years an increasing interest in this approach may also be seen among the noise and vibration specialists (see, e.g., papers [3-5]). In this paper some new results obtained in this field by the author will be presented.

2 SIGNAL PREPROCESSING

Though in the case of nonstationary analysis of the cyclic noise and vibration analog signal processing can also be considered (see, e.g., [6, 7]), digital signal processing has been used almost exclusively nowadays for this purpose.
In the following section digital methods for nonstationary analysis of the cyclic signals will be discussed. All these methods are based on the periodic time averaging and therefore they require that the cyclic noise and vibration signals are sampled at exactly the same number of points in each cycle and these sampling points must correspond to the same phase in different cycles. A time record fulfilling these requirements will be called a cyclostationary signal here.
As far as cyclically working machinery are concerned, a number of suitable preprocessing methods have been developed in the past, mostly in connection with an analysis method known as the order analysis. These methods can also be used either for a direct acquisition of the cyclostationary noise and vibrations records generated by rotating machinery or for transforming the cyclic signals into the cyclostationary ones.
In this respect a well known method is based on using a special electro-optical sensor called a shaft encoder. This sensor is mechanically connected with a suitable revolving shaft of the machinery. The shaft encoder generates pulses corresponding to certain shaft angular positions. The pulses are then fed to control an A/D converter used for collecting the digital data. In this way one is recording the cyclostationary noise and vibration signal directly.
Another method is based on a digital resampling of the analyzed cyclic signal. Now the measured noise and vibration are sampled at a very high constant rate. Concurrently with the data samples tachopulses derived from a machinery shaft rotation are also recorded (usually one tachopulse per shaft revolution). Duration of one cycle and its border points are then exactly defined by two consecutive tachopulses. Using a suitable interpolation technique, new samples within each cycle may be computed. These new samples then represent a cyclostationary signal.
A new technique based on adaptive extraction of the fundamental cyclic component and on determination of its instantaneous frequency has been tested recently. The cyclic noise and vibration record can be then resampled according to this instantaneous frequency. Advantage of this technique is that no special sensor is needed. The results obtained so far are encouraging but the method needs a further refinement to yield results comparable with the first two techniques.
Obviously each of the methods mentioned has its advantages and disadvantages and the best results can be obtained by combining all of them into one preprocessing package.

3 STATISTICAL CHARACTERISTICS

A number of statistical characteristics can be used when analysing cyclostationary signals as nonstationary signals [1]. In the time domain two autocorrelation function can be defined. The first is a double autocorrelation function time-time of a nonstationary signal *x(t)*, which can be computed using the following relation

$$R_{xx}(t_1, t_2) = E[x(t_1)x(t_2)] \quad . \tag{2}$$

Here *E*[ ] denotes expectation.
Similarly a double autocorrelation function time shift-time is defined as

$$R_{xx}(\tau, t) = E[x(t - \tau/2)x(t + \tau/2)] \quad . \tag{3}$$

In the frequency domain one may define another two statistical characteristics. Denoting as *X(f)* the Fourier transform of the signal *x(t)*, a double autospectral density frequency-frequency can be computed using the formula

$$S_{xx}(f_1, f_2) = E[X^*(f_1)X(f_2)] \quad . \tag{4}$$

Here X^* denotes the complex conjugate of *X*. Similarly as in the time domain a double autospectral density frequency-frequency shift can be defined as

$$S_{xx}(f, \alpha) = E[X^*(f - \alpha/2)X(f + \alpha/2)] \quad . \tag{5}$$

A very interesting feature of the nonstationary analysis is a possibility of obtaining time-frequency statistical characteristics of the cyclostationary signals. These can be obtained either from the double autocorrelation functions by a direct Fourier transform or from the double autospectral densities by an inverse Fourier transform. Depending on the characteristics used one obtains either a complex instantaneous autospectrum

$$W_{xx}(f_1, t_2) = \int R_{xx}(t_1, t_2) e^{-j2\pi f_1 t_1} dt_1 \quad , \tag{6}$$

$$W_{xx}(f_1, t_2) = \int S_{xx}(f_1, f_2) e^{j2\pi f_2 t_2} df_2 \quad , \tag{7}$$

or a real instantaneous autospectrum (cyclic Wigner distribution)

$$W_{xx}(f, t) = \int R_{xx}(\tau, t) e^{-j2\pi f \tau} d\tau \quad , \tag{8}$$

$$W_{xx}(f, t) = \int S_{xx}(f, \alpha) e^{j2\pi t \alpha} d\alpha \quad . \tag{9}$$

All these statistical characteristics posses a number of interesting properties and may be conveniently used for particular purposes. However, in the following example of the gear box vibration analysis only the complex instantaneous autospectrum will be considered.

The magnitude of the complex instantaneous autospectrum will be analyzed. As it is a nonnegative function of time and frequency, it is a spectral density. Even more, it can be obtained with any desired time and frequency resolution, it is free of artifacts, contains a useful information on time-frequency signal behavior, and can easily be interpreted.

4 APPLICATIONS TO GEARBOX VIBRATION

As a concrete example, results obtained during a passenger car gearbox testing will be shown here. This example is interesting because it shows extraordinary signal processing requirements encountered in practice. These requirements then led to the development of an algorithm for computing only selected slices from the whole characteristics.

The gearbox tested had 3 shafts, namely input, main, and output shafts. Vibration acceleration signal has been recorded under quasi-constant input shaft speed n_1=3000 min^{-1}, load M=20 Nm, and at the 3rd gear.

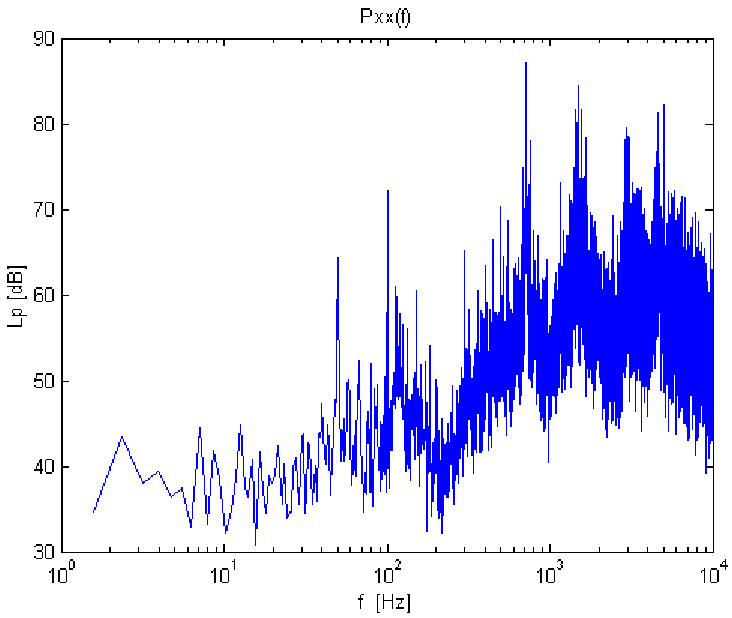

Figure 1: *Autospectral density $P_{xx}(f)$ of the gearbox vibration.*

As the three shafts are mutually interconnected by gear wheels, their cycles are also tied and hence one basic and three shaft cycles are present in the vibration signal. The relation of these cycles is determined by the number of teeth of the respective gear wheels. In the case of the gearbox considered this relation is

$$T_b=95T_{s1}=75T_{s2}=18T_{s3} \quad ,$$

where T_b stands for the basic cycle and T_{s1}, T_{s2}, T_{s3}, are the three shaft cycles.

Under the specified conditions the mean value of the basic cycle was T_b=1.187 s, hence the mean values of the shaft cycles were T_{s1}=19.866 ms, T_{s2}=25.180 ms and T_{s3}=104.930 ms.

The vibration signal was recorded with a signal analyzer whose memory allowed to store 65536 samples. Using the constant sampling frequency of f_s=32768 Hz, the length of the recorded vibration signal was 2 s. Concurrently with the vibration record, tachopulses derived from the input shaft rotation have also been recorded.

First, a stationary approach (continuous time averaging) has been applied to the vibration signal. A standard autospectral density $P_{xx}(f)$ computed in this way is shown in Figure 1. A highly complex structure of the autospectral density can be seen. The autospectrum consists of a great number of discrete spectral components corresponding to the 3 shaft frequencies, and their higher harmonics, 2 toothmeshing frequencies and their higher harmonics, and rich families of sidebands around toothmeshing frequencies.

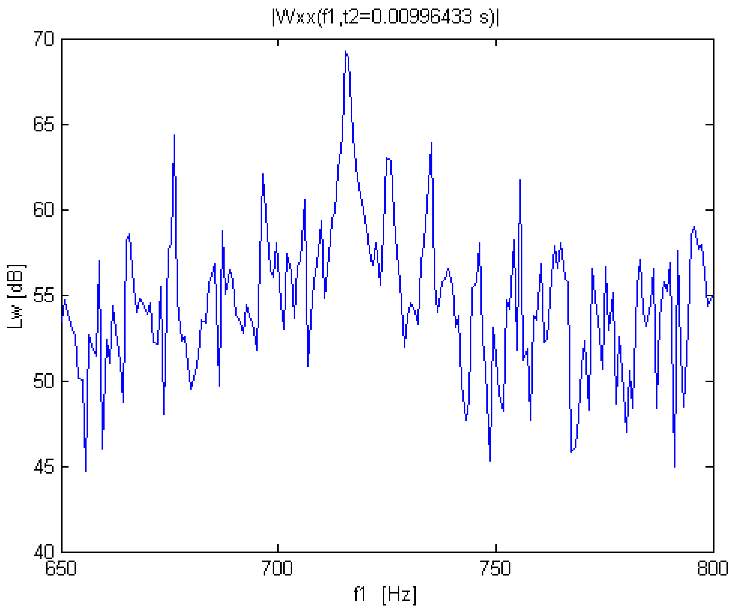

Figure 2: *A selected section from the slice of the complex instantaneous autospectrum* $W_{xx}(f_1,t_2)$.

In the following a result of the time-frequency analysis based on periodic time averaging will be shown. Due to the limited length of the recorded signal, a reasonable number of averages could be obtained only with respect to the shortest shaft cycle, which is T_{s1}. Determination of the whole statistical characteristics within duration of one cycle (0-T_{s1}) and for the basic frequency range (0-f_s/2) would require computation of the double autocorrelation function and of the complex instantaneous autospectrum at as many as 16384x16384 points. This would give a relatively good time resolution Δt=30.772 μs and frequency resolution Δf=0.78711 Hz. However, computation at so many time and frequency points would be very time demanding. Hence, in order to lower the computational burden a procedure for computing only selected slices from the whole complex instantaneous autospectrum has been developed. Even these slices are relatively complex. Therefore in Figure 2 only a section from the slice of the module of the complex instantaneous autospectrum $|W_{xx}(f_1,t_2)|$, computed at the time $t_2\approx$10 ms and in the vicinity of the first toothmeshing frequency, is shown. Again, a very rich structure of the complex instantaneous autospectrum can be seen. The time and frequency resolutions of this characteristics are Δt=30.772 μs and Δf=0.78711 Hz, respectively, and the characteristics is free of artifacts. Thus its interpretation is relatively easy.

5 CONCLUSIONS

The complex instantaneous autospectrum is a powerful tool for time-frequency analysis of cyclostationary signals. Time and frequency resolutions of this characteristics are limited only by hardware available and time one wants to devote to analysis. The characteristics is free of artifacts and can be easily interpreted.

With the real cyclic signals, preprocessing is necessary to obtain cyclostationary records. To overcome an enormous computational burden associated with determination of many element matrices, a method of computing only frequency slices at selected times have been developed. The results of this new technique have been shown using an example of gearbox vibration.

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