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# AMPLITUDES OF FREE BUBBLE OSCILLATIONS IN LIQUIDS

K. VOKURKA†

*Department of Physics, Faculty of Electrical Engineering, Czech Technical University, Suchbátarova 2, CS-166 27 Praha 6, Czechoslovakia*

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In the paper a classification of freely oscillating bubbles is given, methods for evaluating experimental data in research on bubble dynamics are reviewed, and data on bubble oscillation amplitudes, as obtained by analyses of all the accessible experiments (amplitude is defined as the ratio of the maximum to the equilibrium radius of a bubble), is summarized. This shows that the amplitude range of real bubble oscillations is rather limited. This finding, among others, validates the use of simple bubble models for most situations of interest in bubble dynamics research. A comparison with the findings of other workers shows a close correspondence. Finally, suggestions for further research are given.

## 1. INTRODUCTION

Oscillations of bubbles in liquids have been intensively studied for several decades. The reason for this is the ability of bubbles to be harmful in some situations (as, e.g., in the case of cavitation in hydraulic machinery), and to be useful in other situations (as, e.g., in ultrasonic cleaners).

A freely oscillating spherical bubble is described by its size and the intensity of oscillation [1]. It is the intensity of bubble oscillation first of all (but not exclusively) that determines the overall intensity of the processes associated with the bubbles. The intensity of real bubbles' oscillations was recently studied in a series of papers [1-6], where experimental data found in the literature were evaluated. In this paper it is intended to give a classification of oscillating bubbles, to review methods used to evaluate experimental data, and to summarize the data on bubble oscillation intensities as determined in references [1-6].

## 2. CLASSIFICATION OF BUBBLES

Real liquids can always contain oscillating bubbles. These bubbles are produced by a variety of mechanisms, such as transient pressure reduction in the liquid (in this case the phenomenon of bubbles' formation and oscillation is known as cavitation), heating of the liquid (formation of bubbles is known as boiling), underwater chemical and nuclear explosions, electrical discharges in the liquid (spark bubbles), irradiation of the liquid by a focused laser light, blowing a gas or vapour through a nozzle into the liquid, shattering pressurized or evacuated glass spheres in the liquid, etc.

The generated bubbles can be of most diverse shapes and can perform a variety of motions. They can occur in isolation or in clusters (bubble fields), and can be situated either far from the boundaries (extended liquid) or near the boundaries. The bubbles can

† Present address: Švédská 27, CS-466 01 Jablonec n.N., Czechoslovakia.



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perform both free and forced oscillations, and these oscillations can be done in different modes.

In this paper only the simplest case i.e., free radial oscillations of spherical bubbles situated far from boundaries will be considered. (A clarifying note is necessary here: the theoretical results presented here were obtained under the assumption of spherical bubble shape. However, it is well known that experimental bubbles always differ more or less from the ideal spherical shape. This will be also briefly discussed later.) In this section, then, it is intended first of all to classify briefly such bubbles according to their content, method of excitation for free oscillations, intensity of oscillations, and size.

There are two fundamentally different kinds of bubbles: (1) *gas bubbles*, which contain a non-condensable gas, and (2) *vapour bubbles*, which contain a vapour of the surrounding liquid. In a real environment pure gas and pure vapour bubbles occur, most probably, only rarely, and it can be expected that a non-condensable gas is always mixed with a liquid vapour in some ratio, which can vary during the bubble life (due to gas diffusion and vapour evaporation or condensation). Nevertheless it is still convenient to classify the bubbles as being predominantly either gas or vapour ones.

Both the gas and vapour bubbles can be *excited* for free oscillations in three different ways [7]: (i) by increasing bubble energy, (ii) by decreasing bubble energy, and (iii) by temporarily changing the ambient pressure. Whereas the first two methods are, in a way, mutually "symmetric", the third method differs completely.

Depending on the energy supplied during the excitation phase and on the ambient pressure, the excited bubbles can oscillate with different intensities. In this paper the *amplitude* $A = R_M/R_e$ will be used as an oscillation intensity measure. Here $R_M$ is the maximum and $R_e$ the "equilibrium" bubble radius. (The reasons for selecting the amplitude as the oscillation intensity measure are given in the next section.)

According to the intensity of oscillations the gas bubbles can be roughly classified as [8] (a) oscillating *linearly* (approximately for $A \leq 1{\cdot}05$), and (b) oscillating *non-linearly* ($A > 1{\cdot}05$). The case of non-linear bubble oscillations is the most important. (Determination of the oscillation amplitudes in experiments will be the subject of sections 4 and 5.)

Finally, the real bubbles are of different sizes. The *bubble sizes* can be described by the characteristic bubble radii, viz. the maximum, equilibrium, and minimum radii. In this paper the maximum radius, $R_M$, will be used, because it can be determined most easily in experiments. As the bubble size influences strongly the bubble behaviour, it is convenient to classify the bubbles according to their maximum radius in the following way.

(I) *Large bubbles* (macrobubbles). Depending on the amplitude, $A$, the maximum radius of the macrobubbles, $R_M$, is assumed to be larger than 0·01–0·3 m (see Figure 1). For such large bubbles the influence of gravity on the bubble motion cannot be neglected. Bubbles of this size are produced, e.g., by underwater chemical explosions (in this case they can attain maximum radii as large as 10 m [9]), or by underwater nuclear explosions (in this case they can attain maximum radii of the order of $10^2$ m [10]). Evidently, the larger the bubble size, the more pronounced the effect of gravity.

(II) *Small bubbles* (microbubbles). Depending on the amplitude of oscillations, $A$, the maximum radii of these bubbles are assumed to be smaller than $10^{-5}$–$10^{-4}$ m (see Figure 1). For these bubbles the effects of surface tension and viscosity on their motion cannot be neglected. Evidently, the smaller the bubble, the more pronounced are the effects of surface tension and viscosity.

(III) *Medium-size bubbles.* The size of these lies between that of macrobubbles and microbubbles (Figure 1), and they are of greatest interest in this study. A very important subclass of these bubbles are the so-called *scaling bubbles* (Figure 1). For scaling bubbles the effects of gravity, surface tension, viscosity, and heat conduction can be neglected

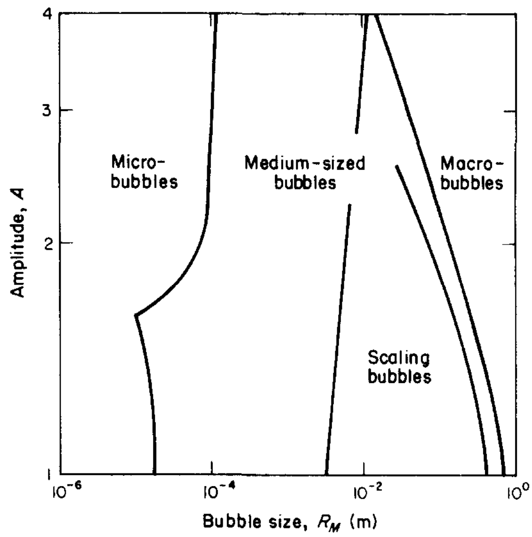


Figure 1. Bubble map (adapted from reference [11]). The map was computed for gas bubbles in water and an ambient pressure equal to the atmospheric one (for other liquids and/or ambient pressures the map will differ from that given here). The limits displayed depend also on the polytropic exponent of the gas in the bubble interior, on the excitation method, etc. (for details see reference [11]).

[11]. It is assumed that the scaling bubbles obey the scaling law [11]. For the non-scaling medium-size bubbles heat losses play an important role. For sufficiently large or sufficiently small bubbles heat losses can be neglected. However, there is a certain range of bubble sizes where heat losses reach a maximum.

## 3. SELECTION OF THE BUBBLE OSCILLATION INTENSITY MEASURE

Besides the bubble size the intensity of bubble oscillations represents the second basic parameter determining the overall bubble behaviour. With respect to a highly diverse world of bubbles a number of oscillation intensity measures can be defined, each being suitable for a particular situation and each having its pros and cons. Unfortunately, none of these measures can directly be determined in experiments (with the exception of a few special cases to be mentioned later). For the purpose of comparison of different bubble excitation techniques it is important to use one measure consistently. Before discussing the amplitude (selected by the author as the intensity measure) let us briefly review some other possibilities.

Probably the most widespread intensity measure is the ratio of the minimum pressure at the bubble wall, $P_m$, and the ambient pressure, $p_\infty$: i.e., $P_m^* = P_m/p_\infty$. This measure is a natural intensity parameter in the compression system of non-dimensional variables [7], which is the favourite system in theoretical studies. This system is also advantageous in experimental work, because the maximum radius, $R_M$, which is a basic parameter in this system, can easily be measured. From the excitation method point of view this system corresponds to the excitation by decreasing the bubble energy. This is, however, an experimental technique used rather seldom. In the case of vapour bubbles this measure can even be a source of confusion, as in this case $P_m = P_{vp}$ ($P_{vp}$ being the liquid vapour pressure), and such a value of $P_m$ does not correspond to the actual intensity of oscillations when compared with the gas bubbles [12]. One can illustrate this by a concrete example. For a vapour bubble oscillating in water, the liquid vapour pressure is $P_{vp} \doteq 2\,\mathrm{kPa}$.

Assuming that the ordinary ambient pressure $p_\infty = 100$ kPa, one obtains $P^*_{mvp} = P_{vp}/p_\infty = 0{\cdot}02$. However, the vapour bubbles oscillate with such an intensity, which in the case of a comparable gas bubble corresponds to a minimum pressure $P^*_{mg} = 0{\cdot}012$ [12].

Another possible intensity measure is the ratio of the maximum pressure at the bubble wall, $P_M$, and the ambient pressure, $p_\infty$: i.e., $P^*_M = P_M/p_\infty$. This is a natural intensity parameter in the expansion system of non-dimensional variables [7]. Unlike the quantity $P^*_m$, this measure can be used directly for comparison of gas and vapour bubbles.

Besides the pressure measures mentioned, there are also intensity measures based on ratios of significant bubble wall positions $R_M$, $R_e$ and $R_m$ ($R_m$ is a minimum bubble radius). Some authors use the ratio $R_m/R_M$ or its inverse $R_M/R_m$. These ratios can be used for comparison of gas and vapour bubbles, but their disadvantage is that they are not natural intensity parameters in any system of variables; i.e., they cannot be substituted directly in any equation of motion.

As said above, the author has selected the ratio $A = R_M/R_e$ and denoted it as the (non-linear) amplitude of oscillations. The reasons for this selection are as follows: first, this ratio is closely related to the measure $P^*_m$ and can thus be directly substituted into the equation of motion in the compression system of the non-dimensional variables ($P^*_m = A^{-3\gamma}$, where $\gamma$ is the polytropic exponent of the gas or vapour in the bubble). Second, the amplitude thus defined corresponds most closely to the notion of the amplitude as used in the field of sound and vibration. Third, in the author's opinion the amplitude $A$ also allows for a quick interpretation and comparison of results in the case of both free and forced oscillations, linear and non-linear oscillations, and gas and vapour bubbles. The disadvantage of this measure is that in the case of a variable ambient pressure, $p_\infty$, the radius $R_e$ has lost its usual meaning, and $A$ has to be determined via some other quantity (e.g., via a peak pressure in the bubble pulse, $p_p$ [5]). In this case the amplitude $A$ represents only an intensity "label", with no direct physical interpretation.

## 4. METHODS FOR EVALUATION OF EXPERIMENTAL DATA

When studying free oscillations of bubbles experimentally, besides the amplitude $A$ and bubble size $R_M$ some further quantities have to be determined, such as the ambient pressure in the liquid, $p_\infty$, which often is time dependent, the polytropic exponent of the gas or vapour in the bubble, $\gamma$, and the properties of the liquid (liquid density $\rho_\infty$, speed of sound in the liquid $c_\infty$, liquid vapour pressure $P_{vp}$, etc.; for a given liquid these constants can usually be found in physical tables).

The *ambient pressure*, $p_\infty$, is either known (e.g., from the configuration of the experiment) or can be measured directly. As far as the *polytropic exponent*, $\gamma$, is concerned, it can in some cases be assumed to be known as well (e.g., if an air bubble is large enough for heat losses to be insignificant and if it oscillates with a moderate intensity so that air can be assumed to behave as an ideal gas, then $\gamma = 1{\cdot}4$). However, very often the value of $\gamma$ is not known accurately. This happens, for example, with bubbles for which heat losses are significant, or for which the intensity of oscillations is so high that the gas cannot be assumed to behave as an ideal one any more, and hence $\gamma \neq$ constant. In these cases one is usually trying to find such an exponent $\gamma =$ constant, which will afford a useful fit of a simplified theory to experimental data. Finally, in some cases the gas inside the bubble is either unknown or its physical properties are not known well. In all these situations $\gamma$ has to be determined experimentally; e.g., indirectly using the method of scaling functions to be discussed later.

The *bubble size*, $R_M$, can be determined either directly by photographing the bubble, or indirectly by measuring the time of the bubble compression or collapse, $T_c$. The time

$T_c$ can be conveniently determined, e.g., from the pressure wave *vs.* time records [1]. The bubble size can be computed from $T_c$ easily if the bubble oscillates either linearly (for $A \to 1$ one has $R_e = (T_c/\pi)(3\gamma p_\infty/\rho_\infty)^{1/2}$, and $R_M = R_e A$ [13]) or with sufficiently intensity (for $A > 3$ one has approximately $R_M \doteq (T_c/0{\cdot}915)(p_\infty/\rho_\infty)^{1/2}$; see, e.g. reference [13]). However, to determine $R_M$ from $T_c$ for other amplitudes one has to use the scaling functions.

Direct determination of the *amplitude*, $A$, is possible in those cases where the equilibrium radius $R_e$ can be measured. This happens when a gas bubble of an initial radius $R_i$ is excited for oscillations by a short pressure pulse [5]. If the ambient pressure returns to the initial value $p_{\infty i}$ fast enough, then $R_e = R_i$ and $R_i$ can be found from bubble photographs taken before the pressure pulse has arrived. If $R_M$, too, is known, the definition formula for $A$ can be directly applied.

In theory $R_e$ might also be determined by photographing the residual bubble at later stages of the bubble life. However, due to rectified diffusion of gases from the liquid into the bubble interior, $R_e$ need not be constant during the bubble life; hence its value determined at later stages cannot be used to compute the amplitude of the first oscillation. Of course, further research is needed to determine the exact role of the rectified diffusion in free oscillations of bubbles.

A theoretical amplitude (i.e., the amplitude the bubble would be excited to oscillate with if the theory were to agree with the experiment) can also be determined from the maximum and minimum pressures at the bubble wall in those cases where these pressures can be measured directly. This is the case, for example, with exploding and imploding glass spheres, where the initial pressure and radius are known. Thus, in the expansion system (pressurized glass spheres [4]) one knows the initial radius $R_{m0}$ and the initial pressure $P_{m0}$. The equilibrium radius can then be computed as $R_e = R_{m0}(P_{M0}/p_\infty)^{1/3\gamma}$ [7]. The maximum radius, $R_{M1}$, can be determined by photographing the bubble. In the compression system (evacuated glass spheres [4]) one knows the initial radius $R_{M1}$ and the initial pressure $P_{m1}$. The theoretical amplitude then is $A_1 = (P_{m1}/p_\infty)^{-1/3\gamma}$ [7].

The pressures $P_{M0}$ and $P_{m1}$ can also be determined from a radiated pressure wave, if one knows $R_{m0}$ and $R_{M1}$, respectively. For example, $P_{M0} = p_{p0}r/R_{m0} + p_\infty$, where $p_{p0}$ is a peak pressure in the "shock wave", and $r$ is a point in the liquid where the wave is measured [4]. In the second case $P_{m1} = p_{v1}r/R_{M1} + p_\infty$, where $p_{v1}$ is a valley pressure in the wave measured at a point $r$ [4].

The theoretical amplitude can also be determined if the gas bubble is excited for free oscillations by a steep ambient pressure increase or decrease to a new value $p'_\infty = p_\infty + \Delta p$, where the pressure change $\Delta p$ lasts for a relatively long time [5]. Then, if the ambient pressure is increased ($\Delta p > 0$), $A_1 = (p_\infty/p'_\infty)^{-1/3\gamma}$ [7], and if it is decreased ($p_\infty - P_{vp} < \Delta p < 0$), $R_e = R_i(p_\infty/p'_\infty)^{1/3\gamma}$ [7], where $R_i$ is the initial bubble radius before the ambient pressure is decreased. Again, in the latter case it is necessary to determine the maximum radius, $R_{M1}$, as well (e.g., by high speed photography).

However, in the majority of situations the above mentioned methods for direct determination of the amplitude cannot be applied, and one has to resort to an indirect method based on the scaling functions.

From what has been said, it follows that when evaluating experimental data one usually wishes to determine two or three parameters describing the bubble behaviour (e.g., $R_M$, $A$ and $\gamma$), and some of them cannot be found directly. A way out of this difficulty is offered by the method of scaling functions. This method is based on the comparison of measured significant bubble wall radii and bubble pulse pressures with suitable scaling functions, which are functional dependences of the measured quantities on the amplitude $A$, exponent $\gamma$ and pressure $p_\infty$. The quantities suitable for comparison are, for example, the minimum radii, $R_m$, compression or collapse times, $T_c$, peak pressures in the bubble pulses, $p_p$, effective widths of the bubble pulses, $\vartheta$, ($\vartheta = (1/p_p^2) \int p_a^2 \, dt$, where $p_a$ is the acoustic pressure in the bubble pulse [1]), radiated acoustic energies, $\Delta E_a$, damping factors, $\alpha$ ($\alpha$ is defined as a ratio of two successive maximum radii; e.g., $\alpha_1 = R_{M2}/R_{M1}$ [1]), maximum wall velocities, $\dot{R}_{max}$, etc.

To reduce the number of variables it is convenient to form non-dimensional pi products of the mentioned quantities. Depending on the situation one may select either the $Z$, $W$, $Y$ or $X$ system of the non-dimensional variables [8]. Then, the non-dimensional minimum radius in the $Z$ system, for example, equals $Z_m = R_m/R_M$, and the corresponding scaling function is $Z_m(A, \gamma, p_\infty)$. It is convenient to display the scaling functions graphically, with $\gamma$ and $p_\infty$ being parameters in these plots.

Ideally the scaling functions should be determined experimentally. However, at present bubble dynamics research is still too far from this situation, for which reason the scaling functions are computed by means of a suitable theoretical model. These computations are performed for the excitation method and liquid of interest (usually for the compression system and water). So far these functions have been determined only for the scaling bubbles (which led, among other things, to their designation "the scaling functions"), and this restricts their application to these bubbles first of all. (One should recall that with scaling bubbles gravity, surface tension, viscosity, and heat conduction are irrelevant.) Several scaling functions taken from reference [1] are displayed in Figures 2–8. Further examples can be found in references [4, 13, 14].

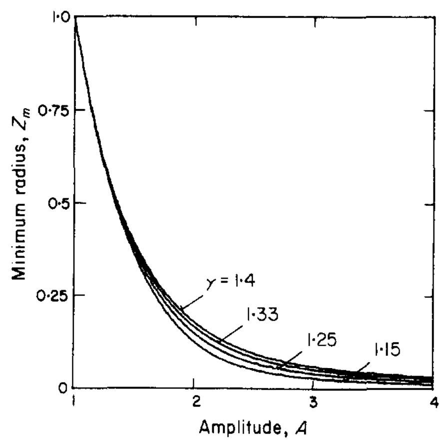


Figure 2. Scaling functions $Z_m(A, \gamma)$ for $p_\infty = 100$ kPa.

A hypothetical example will illustrate the method of the scaling functions best. Let us assume that the maximum radius $R_M$ of a scaling bubble has been determined from bubble photographs. Other quantities also measured are the peak pressure in the bubble pulse, $p_p$, at a point in the liquid, $r$, and a radiated acoustic energy $\Delta E_a$. One wants to find, using the scaling functions, the bubble amplitude, $A$, and the polytropic exponent, $\gamma$. The ambient pressure, $p_\infty$, and the liquid density, $\rho_\infty$, are known. In theory, at least, one has thus sufficient information at one's disposal to solve the problem, because one knows the two non-dimensional variables $p_{zp} = (p_p/p_\infty)r/R_M$ and $\Delta E_{za} = \Delta E_a/(\frac{4}{3}\pi p_\infty R_M^3)$, and one knows also the functional dependences of these variables on the unknown variables $A$ and $\gamma$. In other words, one also knows the functions $p_{zp}(A, \gamma)$ and $\Delta E_{za}(A, \gamma)$, these functions having been computed for the ambient pressure $p_\infty$. Thus, having two functions and two unknown variables the problem can easily be solved; e.g., graphically.

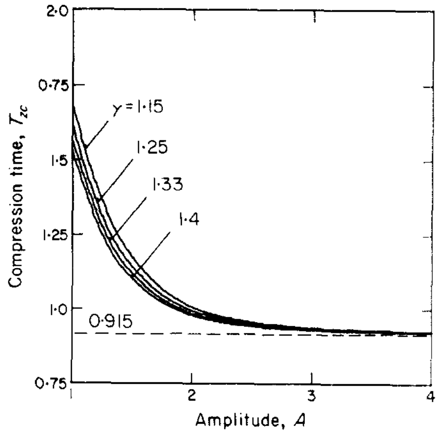


Figure 3. Scaling functions $T_{zc}(A, \gamma)$ for $p_\infty = 100$ kPa.

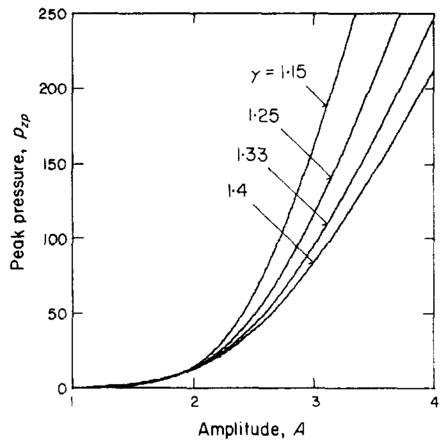


Figure 4. Scaling functions $p_{zp}(A, \gamma)$ for $p_\infty = 100$ kPa.

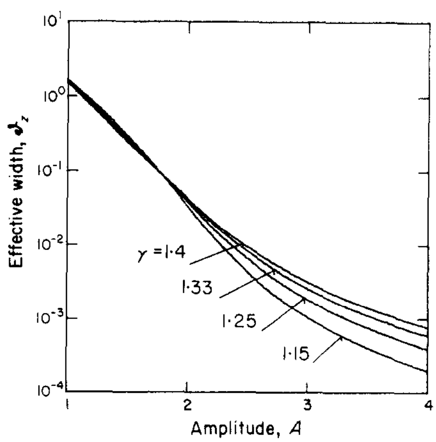


Figure 5. Scaling functions $\vartheta_z(A, \gamma)$ for $p_\infty = 100$ kPa.

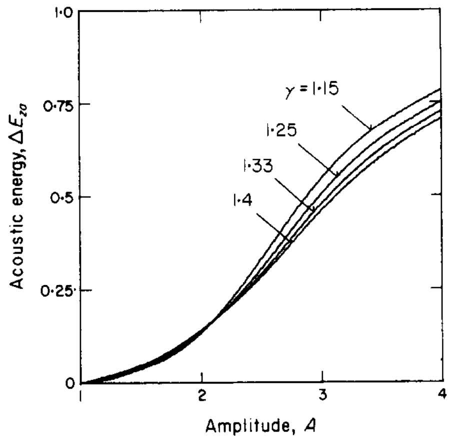


Figure 6. Scaling functions $\Delta E_{za}(A, \gamma)$ for $p_\infty = 100$ kPa.

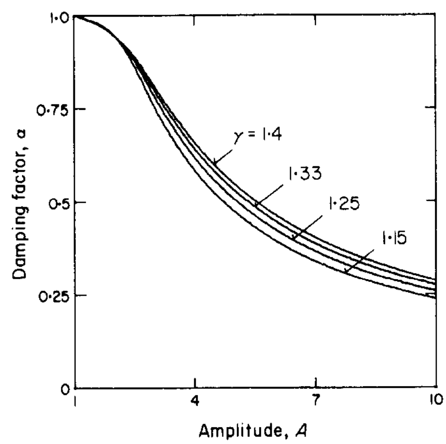


Figure 7. Scaling functions $\alpha(A, \gamma)$ for $p_\infty = 100$ kPa.

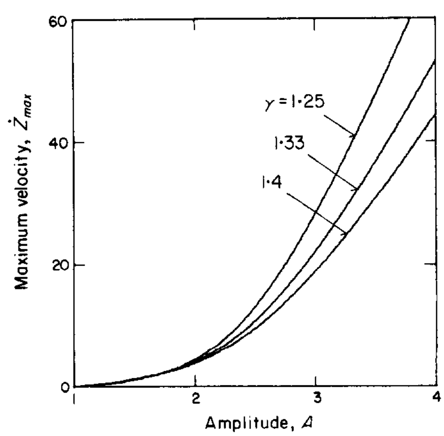


Figure 8. Scaling functions $\dot{Z}_{max}(A, \gamma)$ for $p_\infty = 100$ kPa.

In practice, however, one will encounter serious difficulties when applying this method to experimental data, because at present the theoretical models do not take into account all the energy losses occurring in real bubbles (even such elaborate models as Gilmore's take into account only acoustic radiation; it should be remembered that losses due to viscosity and heat conduction are unimportant for the scaling bubbles). Therefore a measurement of as many quantities as possible is necessary to obtain a more complete picture of what is really happening with the bubble.

When analyzing the data from bubbles generated by underwater explosions [1, 2], it was found that the unaccounted energy losses start acting most probably in the vicinity of $R_{m1}$, and thus it seems that they do not seriously influence the minimum radius, $R_{m1}$, the peak pressure, $p_{p1}$, the effective width of the bubble pulse leading edge, $\vartheta_1'$ [15], the time $T_{c1}$, and the maximum bubble wall velocity during the compression or collapse phases, $\dot{R}_{max\,1}$. These quantities should then be preferred for evaluating the experimental data. On the other hand, the quantities associated with the rebound or expansion phases (e.g., $\alpha$) can be used to assess the unaccounted losses. However, further experiments would be needed to verify these assumptions.

## 5. AMPLITUDES OF BUBBLES' OSCILLATIONS AS DERIVED FROM EXPERIMENTAL DATA

By using the methods briefly described in section 4, a number of experimental data published in the literature were analyzed in references [1-6] with the aim of determining the amplitudes of real bubble oscillations. The bubble sizes *vs.* amplitudes found in this way are summarized in Figure 9 (Figures 1 and 9 should be mutually compared with certain care as they do not always correspond to the same conditions, such as, e.g., $p_\infty$, $\gamma$, method of excitation, etc.). In the following subsections some brief comments and supplementary data are given regarding each excitation technique and each data point displayed in Figure 9. A full description can be found in the source references [1-6].

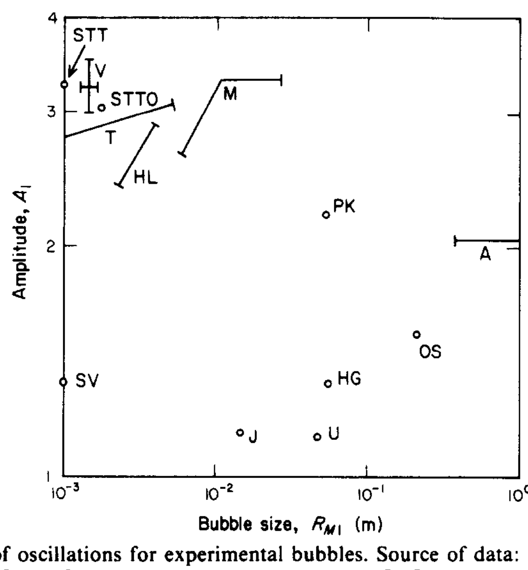


Figure 9. Amplitudes of oscillations for experimental bubbles. Source of data: A, Arons *et al* [17, 18]; M, Mellen [19]; T, Teslenko [20, 21]; HL, Hentschel and Lauterborn [22]; V, Vokurka [15]; STTO, Shima *et al.* [23]; HG, Heuckroth and Glass [24]; PK, Popov and Kogarko [25]; OS, Orr and Schoenberg [26]; U, Urick [27]; SV, Smulders and van Leeuwen [28]; J, Jensen [29]; STT, Shima *et al.* [30].

### 5.1. UNDERWATER CHEMICAL EXPLOSIONS

During underwater chemical explosions the weight of an explosive, $G$, varies from about $10^{-3}$ kg in laboratory tests [16] to $10^3$ kg in the full scale tests [9], and the ambient pressure, $p_\infty$, varies from the atmospheric pressure 100 kPa in laboratory tests [16] to pressures of the order of 10 MPa in deep sea tests. The bubbles produced are typical gas bubbles. In the case of TNT explosives the polytropic exponent is $\gamma = 1{\cdot}25$ [9]. The bubble size $R_M$ varies with $G$ and $p_\infty$ [9], and the amplitude $A$ varies with $p_\infty$ [2]. In a laboratory test, for example, with a TNT charge of a weight $G = 1$ g and an ambient pressure $p_\infty = 100$ kPa, the bubble size and *theoretical amplitude* are $R_{M1} = 0{\cdot}15$ m and $A_1 = 2{\cdot}5$, respectively [1, 2]. By means of the data given in Figure 1 one can see that even this smallest possible explosion generated bubble, when oscillating under atmospheric ambient pressure, does not belong to the class of scaling bubbles.

*Experimental amplitudes* could be determined only in the case of the tests done by Arons *et al.* [17, 18]. In these tests the ambient pressure was higher than the atmospheric one, i.e. $p_\infty = 1{\cdot}68$ MPa, and the weight $G$ of the TNT charges varied from 0·23 kg to 5·4 kg; hence $R_{M1}$ varied from 0·38 m to 1·08 m. The amplitude, as found in reference [2], was $A_1 = 2{\cdot}05$. Using the similarity principle, one can verify that these bubbles should be scaling bubbles [1] (this was actually the reason why the experiments were performed at higher $p_\infty$). These experimental data are shown in Figure 9.

### 5.2. UNDERWATER NUCLEAR EXPLOSIONS

During nuclear explosions extremely high temperatures occur at an epicentre. In the liquid these temperatures cause massive evaporation and a formation of a huge vapour bubble, for which the theoretical amplitude is $A_1 = 3{\cdot}29$ ($\gamma = 1{\cdot}25$) [12].

A description of a bubble produced by an underwater nuclear explosion can be found in reference [10]. According to this report, the initial bubble centre was at the depth $h = 610$ m ($p_\infty = 6{\cdot}2$ MPa), and the bubble grew to a maximum radius $R_{M1} \doteq 115$ m. For a bubble of this size the effect of gravity is very important. Due to it the bubble floats rapidly upwards and loses its spherical shape. Both the vertical motion and non-spherical oscillations cause high energy dissipations, and hence the real amplitude of the bubble oscillations is lower than the theoretical one; i.e., $A_1 < 3{\cdot}29$. Unfortunately, the data given in reference [10] do not allow for any closer estimation.

### 5.3. SPARK AND LASER GENERATED BUBBLES

Experimental data on spark and laser generated bubbles were evaluated in reference [3]. According to this study scaling bubbles were produced only in Mellen's tests [19]. In this case, by using the scaling functions $p_{zp}(A, \gamma)$, it was determined that for bubble sizes, $R_{M1}$, ranging from 10·8 to 27 mm, the amplitude was $A_1 = 3{\cdot}29$ (for $\gamma = 1{\cdot}25$ and $p_\infty = 100$ kPa). For smaller bubbles the amplitude should decrease because of heat losses from the bubble. Unfortunately, functions similar to the scaling functions have not yet been computed for the region of heat losses. However, as a first approximation one can use the functions $p_{zp}(A, \gamma)$ again. The smallest bubble in Mellen's experiments, for example, had a size $R_{M1} = 6$ mm, and the radiated (non-dimensional) peak pressure in the bubble pulse was $p_{zp1} = 65$. Using the scaling functions $p_{zp}(A, \gamma)$ given in Figure 4, one finds that for $\gamma = 1{\cdot}25$ and $p_\infty = 100$ kPa the corresponding amplitude is $A_1 = 2{\cdot}63$.

Other researchers have generated only such bubbles for which heat losses are important. In Teslenko's tests [20, 21], for example, the bubble sizes $R_{M1}$ ranged from 1 to 5 mm, and the peak pressure in the bubble pulse $p_{zp1}$ ranged from 82 to 123 [3]. Hence, using the functions $p_{zp}(A, \gamma)$, one can determine that for $\gamma = 1{\cdot}25$ and $p_\infty = 100$ kPa, $A_1$ ranges from 2·8 to 3·05.

Hentschel and Lauterborn [22] produced bubbles having sizes $R_{M1}$ ranging from 2·2 to 4 mm and peak pressures $p_{zp1}$ ranging from 41 to 100 [3]. Hence using the functions $p_{zp}(A, \gamma)$ one finds that for $\gamma = 1{\cdot}25$ and $p_\infty = 100$ kPa the amplitude $A_1$ ranges from 2·4 to 2·9.

Vokurka [15] worked with bubble sizes $R_{M1}$ ranging from 1·3 to 1·65 mm and amplitudes $A_1$ ranging from 3·0 to 3·5, as found from the bubble pulse leading edge effective widths, $\vartheta_1'$ ($\gamma = 1{\cdot}25$, $p_\infty = 100$ kPa).

Shima *et al.* [23] determined that a spark bubble of a size $R_{M1} = 3{\cdot}5$ mm radiated peak bubble pulse pressure $p_{zp1} = 120$. Hence, the amplitude is $A_1 = 3{\cdot}03$ in this experiment ($\gamma = 1{\cdot}25$, $p_\infty = 100$ kPa). The experimental results mentioned in this subsection are displayed in Figure 9, where the border points mentioned here are connected by straight lines.

It should be noted here that, according to the bubble map given in Figure 1, even Mellen's bubbles having the bubble-sizes $R_{M1}$ ranging from 10·8 to 27 mm and the amplitudes $A_1 = 3{\cdot}29$ are non-scaling bubbles. But this finding must be considered with a certain care. First, the limits of gravity effects displayed in Figure 1 were computed only for $\gamma = 1{\cdot}33$ and, second, the model used for taking the effects of gravity into account was a very rough one. Obviously, further research is needed to derive the limits of the gravity effects more accurately.

For vapour bubbles such as the spark and laser generated ones, the polytropic exponent $\gamma = 1{\cdot}25$ is used rather arbitrarily. At the present state of knowledge it seems that one could just as well use the value $\gamma = 1{\cdot}33$ [12] or any other convenient one. Again, to determine "the most correct" exponent, more complete experimental data are required.

### 5.4. EXPLODING AND IMPLODING HOLLOW GLASS SPHERES

By using hollow glass spheres both gas and vapour oscillating bubbles can be generated. Experimental data on these bubbles have been evaluated in reference [4]. In the following the respective experiments are briefly discussed.

Heuckroth and Glass [24] used an exploding glass sphere pressurized with air ($\gamma = 1{\cdot}4$). The gas bubble grew to a maximum radius $R_{M1} = 0{\cdot}17$ m under the ambient pressure $p_\infty = 100$ kPa. The experimental amplitude, as determined in reference [4], was $A_1 = 1{\cdot}33$.

Popov and Kogarko [25] worked with glass spheres filled with exploding gaseous mixtures. The vapour bubble ($\gamma = 1{\cdot}25$) thus produced grew to a size $R_{M1} = 53$ mm and the experimental collapse amplitude, as stated in reference [4], was $A_1 = 2{\cdot}25$ ($p_\infty = 100$ kPa).

Orr and Schoenberg [26] used an evacuated glass sphere of the size $R_{M1} = 0{\cdot}216$ m. The sphere was filled with air ($\gamma = 1{\cdot}4$) and imploded at an ambient pressure $p_\infty = 30$ MPa. The experimental amplitude was $A_1 = 1{\cdot}55$ [4].

Finally, Urick [27] produced an air bubble ($\gamma = 1{\cdot}4$) of the equivalent radius $R_{M1} = 48{\cdot}3$ mm by imploding a glass bottle under an ambient pressure $p_\infty = 4$ MPa. In this case the experimental amplitude was $A_1 = 1{\cdot}33$ [4]. All these data are displayed in Figure 9.

### 5.5. EXCITATION OF GAS BUBBLES BY CHANGING THE AMBIENT PRESSURE

This excitation technique was used by Smulders and van Leeuwen [28], Jensen [29], Shima *et al.* [30], and Gülhan and Beylich [31]. (Some of the experimental data published by these authors have been evaluated in reference [5].) Smulders and van Leeuwen [28], for example, used an air bubble ($\gamma = 1{\cdot}4$) of a maximum radius $R_{M1} = 1$ mm. The ambient pressure was $p_\infty = 180$ kPa, and the amplitude attained was $A_1 = 1{\cdot}33$ [5]. Jensen [29] used an air bubble ($\gamma = 1{\cdot}4$) of the size $R_{M1} = 15$ mm, and this bubble oscillated under an ambient pressure $p_\infty = 100$ kPa with an amplitude $A_1 = 1{\cdot}15$ [5]. Finally, Shima *et al.* [30] used an air bubble ($\gamma = 1{\cdot}4$) of the radius $R_{M1} = 1$ mm, and this bubble was excited to oscillate with an amplitude $A_1 = 3{\cdot}3$ ($p_\infty \doteq 100$ kPa) [5]. The data are given in Figure 9.

### 5.6. CAVITATION BUBBLES

Experimental data on cavitation bubbles as obtained in tests by Knapp and Hollander [32], Chesterman [33], Schmid [34], Gallant [35], Blake *et al.* [36], Fujikawa and Akamatsu [37], and Bark and van Berlekom [38] have been evaluated in reference [6]. In these experiments the bubble-sizes $R_{M1}$ ranged from 1 to 7 mm, and the ambient pressure $p_\infty$ from approximately 2 to 70 kPa. Unfortunately, from the experimental data available, it was not possible to determine the experimental amplitudes of cavitation bubbles oscillations. However, because both cavitation and spark and laser generated bubbles are vapour bubbles, then assuming that their amplitudes are the same, one obtains $A_1 = 3{\cdot}29$ for $\gamma = 1{\cdot}25$ and $p_\infty = 100$ kPa. However, due to the lower ambient pressures $p_\infty$ in the cavitation tests, the amplitudes may also show other values. For the range of the ambient pressures mentioned the theoretical amplitudes can vary approximately from 3 to 5 [39].

### 5.7. LINEAR OSCILLATIONS OF GAS BUBBLES

Strasberg [40] and Leighton and Walton [41] published experimental data on free linear oscillations of gas bubbles that make it possible to estimate the amplitudes of the oscillations. Strasberg [40], for example, reported that an air bubble ($\gamma = 1{\cdot}4$) of the volume $V_e = 6{\cdot}9 \times 10^{-2}$ cm$^3$ (an equilibrium radius $R_e = 2{\cdot}54$ mm), when leaving a nozzle, produced a decaying sinusoidal pressure pulse with a peak pressure $p_{p1} = 2{\cdot}3$ Pa at a distance from the bubble $r = 1$ m (see Figure 3 in reference [40]). Assuming the ordinary ambient pressure $p_\infty = 100$ kPa, one can calculate the non-dimensional peak pressure [13]: $p_{xp1} = (p_{p1}/p_\infty)r/R_e = 9 \times 10^{-3}$. (Note that for linear oscillations it is convenient to use the $X$-system of non-dimensional variables [8].) Then the "linear" amplitude of bubble oscillations equals [13] $A_{x1} = p_{xp1}/3\gamma = 2{\cdot}2 \times 10^{-3}$. (The "linear" amplitude is defined as $A_x = (R_M - R_e)/R_e$ and is therefore related to the "non-linear" amplitude $A$ by the simple relation $A_x = A - 1$.)

Leighton and Walton [41] reported a similar experiment in which an air bubble ($\gamma = 1{\cdot}4$) of an equilibrium radius $R_e = 1{\cdot}7$ mm produced a peak pressure $p_{p1} = 5{\cdot}5$ Pa at a distance from the bubble $r = 29$ mm. (The value of $p_{p1}$ can be estimated from Figure 5 in reference [41]; the value of $r$ was kindly communicated to the author by Dr Leighton.) Since the ambient pressure is $p_\infty = 1{\cdot}06 \times 10^5$ Pa, one will find that $p_{xp1} = 8{\cdot}8 \times 10^{-4}$ and that $A_{x1} = 2{\cdot}1 \times 10^{-4}$. As the amplitudes of linearly oscillating bubbles are approximately four orders smaller than those of non-linearly oscillating bubbles (expressed in amplitudes $A_x$), they are not given in Figure 9.

## 6. DISCUSSION

Examination of Figure 9 reveals that real bubbles, for which enough experimental data were available to allow determination of the amplitudes, do not oscillate with amplitudes higher than $A \doteq 3{\cdot}5$, the highest amplitudes being found for spark and laser generated bubbles. Unfortunately, the very important class of cavitation bubbles could not be included in Figure 9 because of a lack of suitable experimental data. However, with respect to the collapse mechanism, which should be the same for all vapour bubbles, it can be assumed that the cavitation bubbles oscillate with the same intensity as the spark and laser generated bubbles. As shown in reference [6], the cavitation bubbles usually collapse at low ambient pressures $p_\infty$ (of the order of $10^3$–$10^4$ Pa); and as computed in reference [39], in this area of $p_\infty$ the amplitudes of vapour bubble oscillations can be rather high, ranging, e.g., from 3 to 5. Of course, this theoretical assumption has yet to be experimentally verified.

An interesting question arising from the above discussion is whether there may be bubbles oscillating with amplitudes higher than those shown in Figure 9. The cavitation bubbles just discussed seem to be a case in point. From an analysis given elsewhere [5, 7] it follows that high amplitudes can also be obtained both for gas and vapour bubbles when exciting them for free oscillations by increasing the ambient pressure $p_\infty$. (In experiments this increase of $p_\infty$ is usually accomplished by a shock wave propagated through the liquid containing the bubble). The increased pressure, $p'_\infty$, can last relatively long or can be of short duration (measured by the bubble compression and collapse times $T_c$). In both cases the rise time of the pressure increase, $\Delta p$, is a limiting factor. To obtain high amplitudes the rise time must be much shorter than $T_c$. In the case of excitation by short pressure pulses the most effective excitation is obtained for a square form of the driving pressure pulses and for a driving pressure pulse length $\Delta T$ which is equal to the time $T_c$ [7]. However, practical difficulties associated with producing "square" waveforms of the duration $\Delta T = T_c$ are obvious. A further question to be answered is whether the bubble remains spherical after interaction with the intense pressure pulse.

Unfortunately, so far the potentialities of the method based on the increased ambient pressure have not yet been fully exploited (see Figure 9), and thus it can be said that, with the exception of cavitation bubbles, the vast majority of bubbles oscillate with amplitudes $A < 3{\cdot}5$. This finding implies important consequences for theoretical studies. It follows, for example, that a simple modified Herring's equation, which yields results comparable to the more complex Gilmore's equation for amplitudes as high as $A \leqslant 4{\cdot}5$ [42], can be used to model most bubble situations. In other words, Gilmore's equation has to be resorted to only in a few special cases, such as the initial stages of underwater explosions or excitation of bubbles for free oscillations by strong shock waves, where the oscillation amplitudes are higher than $A > 4{\cdot}5$.

One can also compare the results given in Figure 9 with the findings of other researchers. Shima *et al.* [23], for example, found that an experimental spark bubble of a maximum radius $R_{M1} = 3{\cdot}5$ mm collapsed to a minimum radius $R_{m1} = 0{\cdot}2$ mm, while the maximum pressure at the bubble was $P_{M1} = 200$ MPa. As the ambient pressure was $p_\infty \doteq 100$ kPa, one has $Z_{m1} = R_{m1}/R_{M1} = 0{\cdot}057$ and $P^*_{M1}/p_\infty = 2000$. Benkovskii *et al.* [43] found that an experimental spark bubble of the maximum radius $R_{M1} = 9{\cdot}5$ mm collapsed at a ratio $R_{M1}/R_{m1} \doteq 31$ so that $Z_{m1} = 0{\cdot}032$. These values compare rather well with the results of the present author, who, using Mellen's experimental data [19], has found that the scaling spark bubble collapses to a minimum radius $Z_{m1} = 0{\cdot}0307$ and that the maximum pressure at the bubble wall is $P^*_{M1} = 5460$ (for $\gamma = 1{\cdot}25$) [44].

Another comparison can be made with purely theoretical results. Thus Fujikawa and Akamatsu [45] studied theoretically the effect of the accommodation coefficient for evaporation and condensation, $\alpha_M$, on the attained minimum radius $Z_{m1}$. For a pure vapour bubble of a maximum radius $R_{M1} = 1$ mm they computed the following values of the minimum radius: if $\alpha_M = 0{\cdot}04$, then $Z_{m1} \doteq 0{\cdot}021$; if $\alpha_M = 0{\cdot}01$, then $Z_{m1} \doteq 0{\cdot}05$; and finally if $\alpha_M = 0$, then $Z_{m1} \doteq 0{\cdot}068$ ($p_\infty = 70$ kPa, $\gamma = 1{\cdot}33$). Again, these values of $Z_{m1}$ are of the same order as those obtained in reference [44].

As mentioned in section 2 there is a great variety of oscillating bubbles. Because of the non-linear oscillations and size-dependent effects the bubble behaviour is unique for each amplitude and bubble-size: i.e., one obtains unique bubble radius *vs.* time histories for each $A$ and $R_M$. (An exception are the scaling bubbles where the bubble behaviour is independent of size [11].) Besides, the bubble's behaviour differs for various ambient pressures, $p_\infty$, for various liquids, and in the case of gas bubbles also for various gases in the bubble interior. The bubble behaviour also depends on the vicinity of other bubbles and boundaries, on the excitation technique used, and to a certain degree on some further quantities as liquid temperature, percentage of dissolved gases in the liquid, etc.

To make things even more complicated, after passing the first minimum radius, $R_{m1}$, the real bubble often starts behaving in a random manner; i.e., even two bubbles having initially the same size, amplitude, etc., may attain different second maximum bubble radii, $R_{M2}$. This is due, most probably, to a random distortion of the initially spherical form when the bubble wall passes through $R_{m1}$. It can be assumed that the distortion induces a turbulence in the liquid and that this turbulence is responsible for "excessive" energy losses. To describe the random wall distortion, a new parameter, distortion factor $\varepsilon$, has been suggested in reference [4].

Thus the spectrum of real bubble behaviours is immensely broad which makes a comparison of different experimental results, a search for some order in the data, and finally a generalization of the data, a very difficult task. This difficulty is further increased by the lack of suitable experimental data. To make some progress in this extraordinarily complex field, research on bubble dynamics should start with the simplest possible case, which is an isolated, spherical, scaling bubble, oscillating (pulsating) freely and radially. A thorough understanding of the behaviour of such a bubble will then form a suitable basis for studying more complicated cases, such as non-scaling bubbles, bubbles oscillating near boundaries, and bubbles performing forced oscillations. (One should bear in mind that inclusion of any one of these effects adds another dimension to the overall complexity.)

There are some further reasons why the scaling bubbles represent a very important class of bubbles. Not only is the analysis substantially simplified by excluding the effects dependent on bubble size, but their size is also very convenient from the point of view of experimental data acquisition. For larger bubbles, for example, a higher time resolution can be obtained, and thus finer details can be examined with a given camera framing rate, just because the larger bubbles move more slowly (the time of the bubble oscillation being directly proportional to its size). Similar considerations apply to the frequency response of hydrophones and associated electronic apparatus [1]. These conveniences of larger bubbles may seem to be more than obvious, but examination of the literature reveals that experimenters do not exploit them very often. (An exception known to the author is some work mentioned by Blake and Gibson [46].)

There are, of course, also upper limits regarding the optimum bubble size. These follow from the effect of gravity on large bubbles, from the energies involved in generating large bubbles, and finally from requirements as to the dimensions of the test tank. When taking all this into account, it seems that the optimum bubble size really coincides with that of the scaling bubble.

As discussed in detail in reference [1], as many quantities as possible should be measured if one wants to arrive at a solid interpretation of the experimental data in bubble dynamics studies. The measured quantities can be, for example, a succession of the maximum bubble radii $R_{M1}, R_{M2}, \ldots$, of the collapse or compression times $T_{c1}, T_{c2}, \ldots$, of the peak pressures in the bubble pulses $p_{p1}, p_{p2}, \ldots$, and of the effective widths of the bubble pulses $\vartheta_1, \vartheta_2, \ldots$. To obtain all these data, the bubble wall motion and the radiated pressure waves have to be monitored at the same time. This should be done simultaneously both with high framing/digitizing rates during short time intervals in the vicinity of $R_m$s and $p_p$s to catch the details of these highly interesting regions, and with lower framing/digitizing rates during long time intervals spanning several bubble oscillations to register the bubble's overall behaviour. The ambient pressure $p_\infty$ should always be monitored. Unfortunately, such thorough investigations are very rare. Most authors content themselves with a tiny fraction of the suggested measurements.

A good deal of information can also be obtained by using simpler experimental set-ups. Probably the simplest possible approach is by means of a hydrophone and several digital event recorders to monitor the times of spark and laser generated bubble oscillations $T_{01}, T_{02}, \ldots$, the details of the bubble pulses, such as the effective bubble pulse widths $\vartheta_1, \vartheta_2, \ldots$, (in this case the hydrophone even need not be calibrated [15]), and the peak pressures $p_{p1}, p_{p2}, \ldots$. Another very simple method is to monitor by high speed photography (or by an equivalent optical method) the maximum radii $R_{M1}, R_{M2}, \ldots$. However, it should be stressed here that it is totally insufficient to monitor only one bubble oscillation, because the information obtained—a "reversed cup" form of the bubble radius *vs.* time record—has been known for more than 60 years and tells us nothing new.

The measurements mentioned should be performed with relatively large ensembles of bubbles of different sizes (covering as large ranges as possible), for different ambient pressures, $p_\infty$, liquids, excitation techniques, and in the case of gas bubbles also for different oscillation amplitudes and gases in the bubble interior. Thus the volume of the work to be done is enormous. But only when this work is performed will solid foundations for further research on bubble dynamics have been laid.

To conclude, let us say a few words about the mathematical models of bubbles. In the literature a broad spectrum of bubble models has been described. At one end of it there are models as simple as Rayleigh's models of empty or gas bubbles. These models are rather crude and are used only as first approximations. At the other end of the spectrum there are highly elaborate models, the equations of which take several pages to print. Writing computer programs for these models and running these programs on a computer is very time-demanding. Small wonder that these advanced models are by and large used only by their inventors (and even they do so rather limitedly) and that experimenters prefer using simpler models.

As shown above, the range of real bubbles and the physical conditions under which bubbles exist is extremely broad. The non-existence of analytical solutions of bubble models implies therefore that in order to obtain more general results one has to vary the different physical parameters in the equations and to repeat the numerical integration for each new value of the parameters. This would mean an enormous volume of computations. Evidently, therefore, bubble models are needed that are both simple enough (to keep the computation time down at an admissible level) and sufficiently accurate (to obtain results one can trust). However, it is difficult to meet both requirements at the same time. Thus compromises will have to be made.

To cope with this problem the author used several simple models [1, 2, 5, 6, 11, 12, 14, 39, 44] that proved to be both computationally effective and (in a sense) accurate. These models do not pretend to compete with the advanced models used in theoretical studies. But only by using them was it possible to perform the hundreds of numerical integrations that are needed to construct the scaling functions and other bubble characteristics and to obtain an insight into the bubble behaviour.

## 7. CONCLUSION

In this paper a classification of oscillating bubbles has been given, methods for evaluating experimental data in bubble dynamics have been reviewed, and data on bubble oscillation amplitudes have been summarized (these data having been obtained by analyses of all accessible experiments on bubble dynamics). It is hoped that by summarizing these scattered results a novel and, in a way, a unifying view on bubbles' oscillation intensities has been obtained.

It follows from Figure 9 that, in contrast to opinions sometimes held in the literature, the range of real bubble oscillation intensities is rather limited. This finding accords with other observations. For example, when analyzing the data published in the literature, one can see that there is no real experimental evidence for (in the theoretical literature so often mentioned) "supersonic velocities of the bubble wall", "shock formations in the bubble pulse", etc., these phenomena being associated with extremely intense (and thus, as shown here, unrealistic) bubble oscillations. However, one can note that even with the intensities limited in the sense mentioned above, oscillating bubbles are remarkable dynamic systems performing highly non-linear oscillations hardly to be found elsewhere in nature.

It was also stressed that the spherical bubble shape, taken for granted in many studies (including some by the present author), is only a very rough approximation to reality. Even bubbles that are initially spherical lose their spherical form at later stages (often in the vicinity of the minimum radius), and the smooth wall becomes distorted. The distortion then induces turbulence in the surrounding liquid and enlarges the bubble surface, thus increasing the energy losses from the bubble.

In this paper, the amplitudes of only free bubble oscillations have been considered, and no attempt has been made to do the same for forced bubble oscillations. The reason for this is that forced oscillations are much more complicated and at the same time much less amenable to both theoretical and experimental studies than free ones, and so far the experimental evidence gathered is very limited. In the author's opinion a real breakthrough in understanding forced bubble oscillations will be obtained only after the free oscillations have been thoroughly understood.

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