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INFLUENCE OF THE AMBIENT PRESSURE ON FREE OSCILLATIONS OF BUBBLES IN LIQUIDS†

K. VOKURKA‡

Department of Physics, Faculty of Electrical Engineering, Czech Technical University, Suchbátarova 2, CS-166 27 Praha 6, Czechoslovakia

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Theoretical results demonstrating the influence of the ambient pressure on free oscillations of gas and vapour bubbles in liquids are presented. Acoustic losses, which increase with the ambient pressure, are shown to be a major factor influencing the overall bubble behaviour. The analysis reveals interesting facts about vapour bubbles, for which a minimum of the oscillation intensity is found at a certain ambient pressure.

1. INTRODUCTION

In previous papers [1-3] free oscillations of bubbles in liquids were analyzed under the assumption that the ambient pressure, p_∞, equalled the atmospheric pressure, i.e., $p_\infty = 10^5$ Pa. Extensive computations were carried out for this basic condition and the results obtained were used in evaluating a number of interesting experimental data [4].

However, in many experiments the ambient pressure in the liquid differs substantially from the atmospheric pressure. For example, in cavitation test tunnels the ambient pressure during the bubble collapse is usually of the order of 10^4 Pa [5-8]. In experiments with bubbles formed by a passage of a tensile pulse through the liquid, the bubbles usually collapse at an ambient pressure which is only slightly higher than the liquid vapour pressure, P_v, i.e., p_∞ is of the order of 10^3-10^4 Pa [9-12].

In the papers mentioned above the ambient pressure was lower than the atmospheric pressure. However, experiments have also been recorded in which the ambient pressures were higher than the atmospheric pressure. For example, in investigations described by Smulders and van Leeuwen [13] the ambient pressure ranged from $1{\cdot}25 \times 10^5$ to $2{\cdot}9 \times 10^5$ Pa during the compression phase. Blaik and Christian [14] reported measurements relating to bubbles formed at great depths in the ocean by underwater explosions. In this case the ambient pressure was of the order of 10^7 Pa. Similarly, Urick [15] and Orr and Schoenberg [16] produced oscillating gas bubbles by smashing partially evacuated glass spheres at great depths in the ocean. Again, the ambient pressure was of the order of 10^7 Pa. Finally, Vorotnikova *et al.* [17] worked with a shock tube that made it possible to excite gas bubbles for free oscillations by high and long pressure pulses. In these experiments the ambient pressure ranged from 10^7 to 10^9 Pa.

Unfortunately, due to sound radiation, exact scaling is not possible between bubbles oscillating at different ambient pressures even for medium-sized bubbles. Hence the results presented in references [2, 3] could be used to analyze the bubble behaviour under the new ambient pressure conditions only with the utmost care. It seems reasonable therefore

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‡ Present address: Department of Research and Development, o.p. LIAZ, tř. V. Kopeckého 400, CS-466 05 Jablonec n.N., Czechoslovakia.

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to examine the influence of the ambient pressure on free oscillations of bubbles in greater detail.

In this paper some results of numerical computations are presented, which, it is hoped, will throw more light on this problem. To keep the exposition as simple as possible, only medium-sized bubbles, for which the effects of gravity, surface tension, viscosity, and heat conduction can be neglected, will be discussed [1, 18]. The bubble form is assumed to be spherical throughout. The bubble wall motion will be described by Herring's modified equation of motion which, in spite of its simplicity, yields reasonably accurate results [19]. The excitation for free oscillations [20], although it determines the overall bubble behaviour, will not be considered; rather the bubble wall motion will be followed, starting at the first maximum radius, R_{M1}. Finally, the ambient pressure, p_∞, is assumed to be constant during the analysis. (A list of nomenclature is given in the Appendix.)

2. BUBBLE MODEL

At present several equations describing the bubble wall motion are used in the literature. Among them an equation due to Gilmore gives probably the most satisfactory results for the broadest range of bubble oscillation intensities. However, as shown elsewhere [19], for low and medium intensities of bubble oscillations a much simpler Herring's modified equation of motion also gives reasonably accurate results (as compared to Gilmore's model) with considerably less demand on computing time. As the range of intensities for which Herring's modified equation can be used is sufficient for the present problem it will be used in this paper to save computing time. This equation of motion can be written as [19]

$$\ddot{R}R+\tfrac{3}{2}\dot{R}^2=(1/\rho_\infty)[P-p_\infty+\dot{P}R/c_\infty]. \tag{1}$$

Here R is the bubble radius, ρ_∞ the liquid density, P the pressure in the liquid at the bubble wall, and c_∞ the speed of sound in the liquid. The overdots denote differentiation with respect to time.

An oscillating bubble radiates pressure waves into the surrounding liquid. If the propagation of these waves can be treated in the frame of linear acoustics, then the peak pressure in the first bubble pulse, p_{p1}, at a point r in the liquid, can be expressed as [1, 2]

$$p_{p1}=(P_{M1}-p_\infty)R_{m1}/r. \tag{2}$$

Here P_{M1} is the (maximum) pressure at the bubble wall when $R=R_{m1}$, and R_{m1} is the first minimum bubble radius.

In the case of a gas bubble the pressure at the bubble wall equals [1-3]

$$P=P_{m1}(R_{M1}/R)^{3\gamma}. \tag{3}$$

Here P_{m1} is the (minimum) pressure at the bubble wall when $R=R_{M1}$, and γ is the polytropic exponent of the gas in the bubble.

In the case of a vapour bubble a simple model described in detail in reference [3] will be used. This model is based on an assumption that at first, when the bubble wall velocity is low, condensation and evaporation keep pace with the bubble wall motion thus maintaining the pressure inside the bubble approximately equal to the liquid vapour pressure P_v. At later stages, when the bubble wall velocity increases above a certain velocity, $\dot{R}_{vg}$, it is assumed that evaporation or condensation are insignificant. Hence the pressure at the bubble wall can be expressed at a first approximation as [3]

$$P=\begin{cases} P_v & |\dot{R}|<|\dot{R}_{vg}|, \quad R>R_{vg} \\ P_v(R_{vg}/R)^{3\gamma}, & |\dot{R}|\geqslant|\dot{R}vg|, \quad R\leqslant R_{vg} \end{cases}. \tag{4}$$

Here R_{vg} is the radius at which the bubble changes its behaviour. For $R > R_{vg}$ the bubble is assumed to behave as an *ideal vapour bubble* (at which condensation and evaporation take place at an infinite speed), and for $R \leqslant R_{vg}$ to behave as an *ideal gas bubble* (at which condensation and evaporation are zero). The value of the transition velocity, R_{vg}, was determined in reference [3] by comparing an experimental peak pressure in the first bubble pulse, p_{p1}, with a computed one.

That part of the vapour bubble collapse time, T_c, during which the bubble behaves as the ideal vapour bubble, will be denoted as ΔT_v, and the part where it behaves as the ideal gas bubble as ΔT_g. Evidently, $T_c = \Delta T_v + \Delta T_g$. The meaning of the terms just introduced, and of some further ones, is shown in Figure 1.

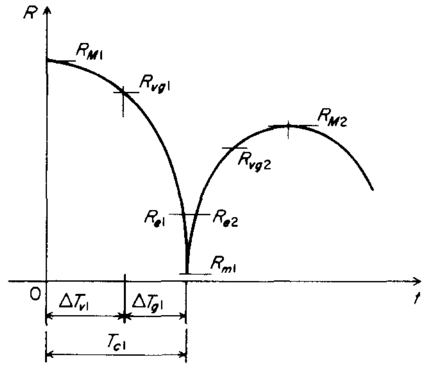

Figure 1. Time history of the vapour bubble wall motion: definition of the basic quantities.

At this stage of the discussion it is convenient to introduce *non-dimensional variables*. In this paper the compression system will be used, in which the non-dimensional quantities are defined as follows [1, 2, 20]:

$$t_z = t/[R_{M1}(\rho_\infty/p_\infty)^{1/2}], \qquad Z = R/R_{M1}, \quad P^* = P/p_\infty,$$
$$p_{zp1} = (p_{p1}/p_\infty)r/R_{M1}, \qquad c^*_\infty = c_\infty(\rho_\infty/p_\infty)^{1/2}.$$

Thus equations (1)-(4) take the forms

$$\ddot{Z}Z + \tfrac{3}{2}\dot{Z}^2 = P^* - 1 + \dot{P}^* Z/c^*_\infty, \tag{5}$$

$$p_{zp1} = (P^*_{M1} - 1)Z_{m1}, \qquad P^* = P^*_{m1}Z^{-3\gamma}, \tag{6, 7}$$

$$P^* = \begin{Bmatrix} P^*_v, & |\dot{Z}| < |\dot{Z}_{vg}|, & Z > Z_{vg} \\ P^*_v(Z_{vg}/Z)^{3\gamma}, & |\dot{Z}| \geqslant |\dot{Z}_{vg}|, & Z \leqslant Z_{vg} \end{Bmatrix}. \tag{8}$$

The *intensity* of bubble oscillations may be expressed in several ways [20]. For gas bubbles oscillating in the compression system the natural intensity measure is the initial gas pressure, P^*_{m1}. However, for the purpose of comparison of different excitation techniques and for comparison with vapour bubbles it is convenient to use the amplitude of oscillations, $A_1 = R_{M1}/R_e$, where R_e is the equilibrium radius defined by the condition that $P^* = 1$ when $R = R_e$ [1]. In the simple case considered here (equation (7)) one has $A_1 = (P^*_{m1})^{-1/3\gamma}$.

In the case of vapour bubbles the intensity measure P^*_{m1} cannot be used [3], and it is the amplitude A_1 which now represents a suitable measure. However, now the "equilibrium radius" has lost its meaning as a radius the bubble wall approaches when $t_z \to \infty$ [21]. Nevertheless, the equilibrium radius can still be defined (and computed) from the condition that $P^* = 1$ when $Z = Z_e$. Note that in the compression system one has $A_1 = 1/Z_e$.

3. RESULTS

Qualitative information on *gas bubble* behaviour at different ambient pressures can be obtained in the following way. Consider for a moment a bubble oscillating in a non-compressible liquid (i.e., Rayleigh's model [1, 20]). In this case, and only in this case, scaling between different ambient pressures is possible. (Scaling means that results obtained at one ambient pressure can be transformed to other ambient pressures by a simple change of the scales by which the respective quantities are measured [1, 18]).

The quantities of interest in the investigation under discussion are the wall velocity $\dot{R}$ and the acceleration $\ddot{R}$. In terms of the non-dimensional solution of Rayleigh's model these quantities can be expressed as $\dot{R} = \dot{Z}(p_\infty/\rho_\infty)^{1/2}$ and $\ddot{R} = \ddot{Z}(p_\infty/\rho_\infty)$. From these relations one can see that when the value of p_∞ is increased while the values of $\dot{Z}$, $\ddot{Z}$ and ρ_∞ are kept constant, the wall velocity and acceleration also increase. However, as the radiated pressure wave is directly proportional to $\dot{R}$ and $\ddot{R}$ [1], the radiated acoustic energy also increases with p_∞. Evidently, bubbles with different acoustic losses are not similar, and scaling therefore, as anticipated, is really not possible. But it also follows that in more appropriate bubble models, the non-dimensional first minimum radius, Z_{m1}, and the time of the first bubble compression, T_{zc1}, must increase with p_∞ due to increased acoustic losses. On the other hand, the maximum pressure, P^*_{M1}, the peak pressure in the bubble pulse, p_{zp1}, and the second maximum radius, Z_{M2}, must decrease with p_∞. As will be seen later, this simple qualitative analysis fully accords with numerical results.

The behaviour of *vapour bubbles* is, due to the collapse mechanism [3], not as clear as that of the gas bubbles. A simple qualitative analysis similar to the one presented above is thus not possible. However, a detailed discussion will be given later on.

3.1. GAS BUBBLES

Equations (5)-(7) were solved for different values of the initial pressure, P^*_{m1}, and of the ambient pressure, p_∞. For each computer run the following quantities were recorded: Z_{m1}, T_{zc1}, P^*_{M1}, p_{zp1}, and α_1, the damping factor, which in the compression system equals the second maximum radius, i.e., $\alpha_1 = Z_{M2}$ [2]. Variations of these quantities with p_∞ are shown in Figures 2-6. Note that the initial gas pressure P^*_{m1}, which is a parameter in these plots, represents an intensity of bubble oscillations (as shown above, P^*_{m1} can easily be converted to the amplitude A_1), i.e., the individual curves in Figures 2-6 correspond to constant amplitudes A_1. Computations were performed for water ($\rho_\infty = 10^3$ kg m^{-3}, $c_\infty = 1450$ m s^{-1}) and for a gas having polytropic exponent $\gamma = 1{\cdot}25$. As can be seen from Figures 2-6, the numerical results do conform with the simple qualitative analysis presen-

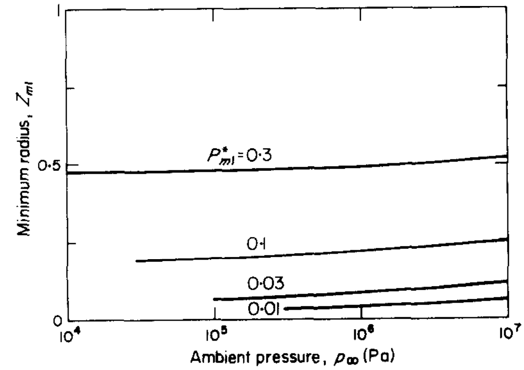

Figure 2. Variation of the first minimum radius, Z_{m1}, with the ambient pressure, p_∞, for gas bubbles.

ted above. It may be added that the quantities T_{zc1}, p_{zp1} and α_1 can be determined experimentally and can thus be used to validate the present theories.

3.2. VAPOUR BUBBLES

In this case equations (5), (6) and (8) were solved for different values of the ambient pressure p_∞. For each computer run the same quantities as above were recorded. For

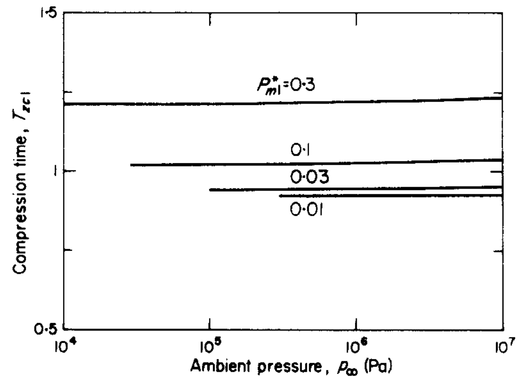

Figure 3. Variations of the first compression time, T_{zc1}, with the ambient pressure, p_∞, for gas bubbles.

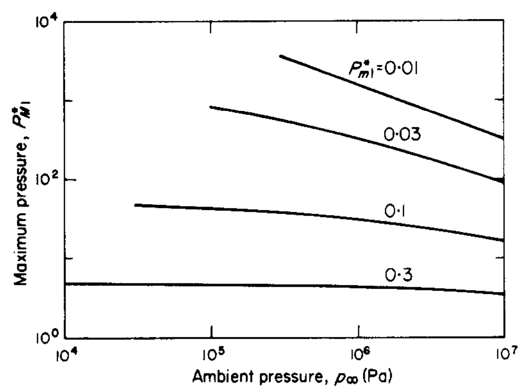

Figure 4. Variation of the first maximum pressure, P^*_{M1}, with the ambient pressure, p_∞, for gas bubbles.

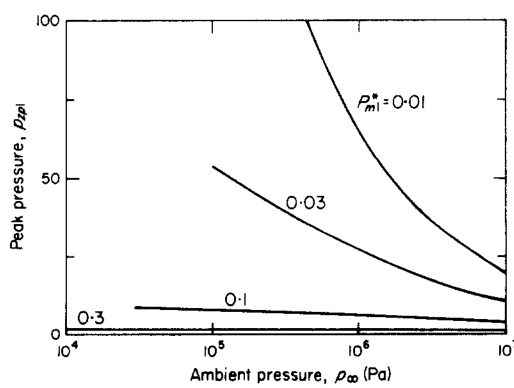

Figure 5. Variation of the peak pressure in the first bubble pulse, p_{zp1}, with the ambient pressure, p_∞, for gas bubbles.

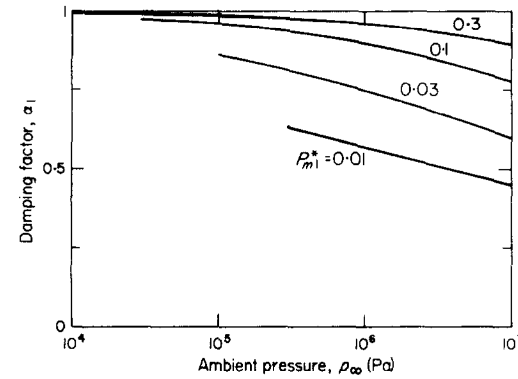

Figure 6. Variation of the first damping factor, α_1, with the ambient pressure, p_∞, for gas bubbles.

reasons given later it was also found useful to record the values of ΔT_{zv1}, ΔT_{zg1} and Z_{e1}. The computations were performed again for water ($P_v = 2$ kPa, $|\dot{R}_{vg}| = 6\ \mathrm{m\,s^{-1}}$, $\gamma = 1·25$ [3]). Variations of the computed quantities with p_∞ are shown in Figures 7–12.

It is possible now to give an *interpretation* of the results presented in Figures 7–12. First, it should be recalled that the individual curves shown in Figures 2–6 correspond

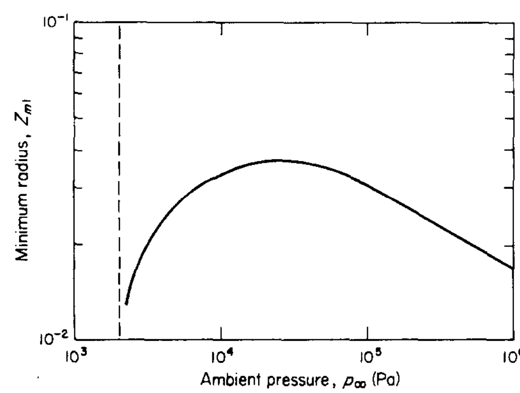

Figure 7. Variation of the first minimum radius, Z_{m1}, with the ambient pressure, p_∞, for vapour bubbles.

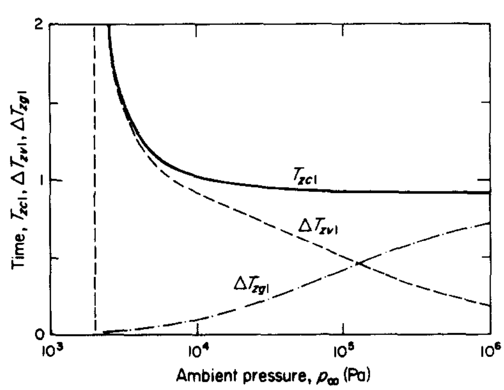

Figure 8. Variation of the first collapse time, T_{zc1}, and of the times ΔT_{zv1}, ΔT_{zg1}, with the ambient pressure, p_∞, for vapour bubbles.

to the constant amplitudes of bubble oscillations and that for a particular p_∞ the gas bubbles can be excited to oscillate with different amplitudes A_1. In contrast to this the vapour bubbles can oscillate with only one amplitude A_1 at a given ambient pressure p_∞ and this amplitude varies with p_∞ (cf. Figure 12). This can be explained as follows.

From Figure 8 one can see that at *low ambient pressures* p_∞, the time ΔT_{zv1}, during which the bubble behaves as an ideal vapour bubble, increases if the pressure p_∞ is

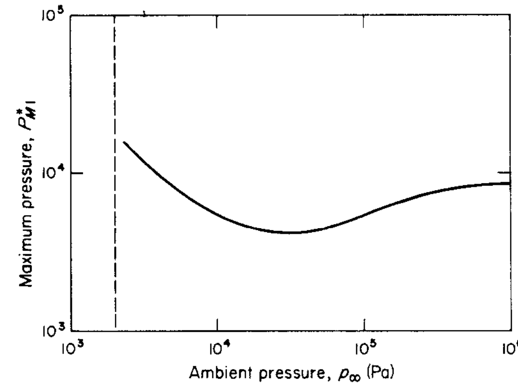

Figure 9. Variation of the first maximum pressure, P^*_{M1}, with the ambient pressure, p_∞, for vapour bubbles.

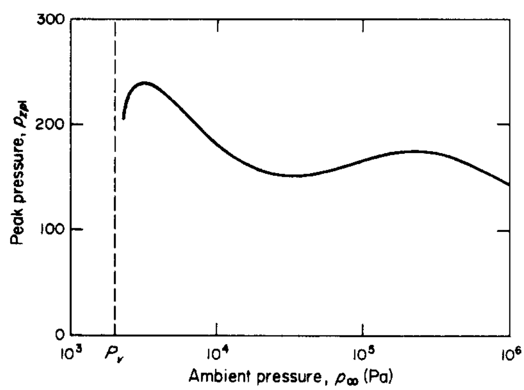

Figure 10. Variation of the peak pressure in the first bubble pulse, p_{zp1}, with the ambient pressure, p_∞, for vapour bubbles.

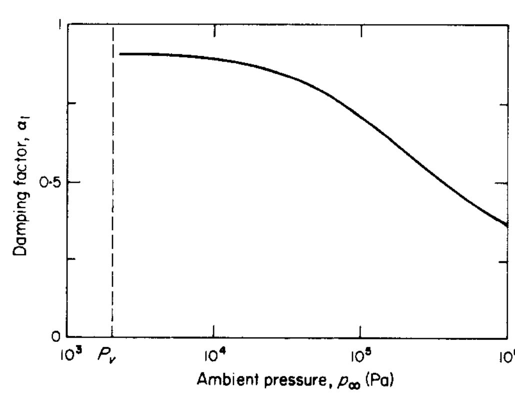

Figure 11. Variation of the first damping factor, α_1, with the ambient pressure, p_∞, for vapour bubbles.

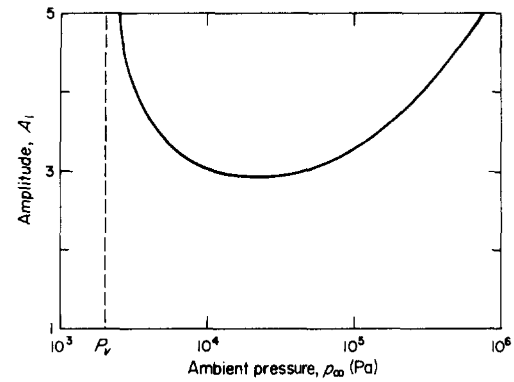

Figure 12. Variation of the first amplitude of the vapour bubble oscillation, A_1, with the ambient pressure, p_∞.

decreased. Recall here that for the ideal vapour bubble the pressure at the bubble wall equals $P = P_v$ [3]. If $P = P_v$ during the whole collapse phase, one obtains Rayleigh's model of an "empty" bubble for which $A_1 \to \infty$ [1]. Thus the larger the portion of T_{zc1} that goes to ΔT_{zv1}, the more the bubble behaves as an empty bubble and the larger the amplitude A_1.

On the other hand, at ***high ambient pressures*** p_∞, the time ΔT_{zg1}, during which the bubble behaves as an ideal gas bubble, is much larger than ΔT_{zv1}. Thus the bubble behaves as a gas bubble during most of its life. Then keeping P_v constant while increasing p_∞ causes the initial pressure, $P^*_{m1} \sim P_v/p_\infty$, to be decreased, and hence the amplitude A_1 to be increased. Note that at ***medium pressures*** p_∞, the times ΔT_{zv1} and ΔT_{zg1} are approximately the same and the amplitude A_1 is minimum. This situation occurs for pressures approximately $p_\infty = 30$ kPa.

From the foregoing discussion it follows that for vapour bubbles the changes in p_∞ also influence the value of A_1. Apart from that, it is known that both the pressure p_∞ and the amplitude A_1 have direct influence on the acoustic losses. For higher pressures p_∞ these losses grow due to the increase in both the amplitude A_1 and the influence of p_∞ on the acoustic radiation (see Figure 6). For low pressures p_∞ the losses decrease due to the influence of the pressure p_∞ on the acoustic radiation and the inherently low acoustic radiation of the ideal vapour bubble. (In the ideal vapour bubble, acoustic radiation results from the dynamic motion of the bubble wall only, and this motion is rather slow for most of the collapse phase, i.e., $|\dot{R}| < 6\ \mathrm{m\,s^{-1}}$. There is no pressure wave component communicated from the vapour into the liquid [1]. Thus the larger ΔT_{zv1}, the smaller the radiated energy.)

The particular features of the curves displayed in Figures 7–11 are thus determined by a combination of the effects mentioned above. In some situations it is the increase of the amplitude A_1 that plays the decisive role; in others the effect of acoustic losses may prevail.

4. DISCUSSION

In this paper a simple bubble model based on the modified Herring's equation of motion (1) and on the assumption of an ideal gas (equations (3) and (4)) has been used. By using Gilmore's equation of motion [19] and a two-gamma equation of state for the gas [22] it would certainly be possible to improve the accuracy of computations at higher pressures. However, in the author's opinion, such improvements do not carry far. As shown elsewhere [2, 4, 22], as well as acoustic losses there are further energy losses even for medium-sized bubbles. These losses may account for as much as 50% of the total dissipated energy during the first compression and expansion phase. A presence of these losses in real bubbles was determined by a detailed breakdown of all the known energies associated with the bubble oscillations and by comparing the theoretical and experimental data (for details see reference [2]). Recent investigations carried out by the author indicate that these losses have a universal character, i.e., they are present in any kind of non-linearly oscillating bubbles. Unfortunately, the nature of the losses has not been clarified yet. In the literature various possible dissipative mechanisms have been suggested. Among them are [22]: (i) turbulence induced in the water surrounding the bubble, (ii) loss of gas from the main bubble, (iii) excessive cooling in the protuberances, and (iv) internal converging shocks.

As the unknown losses amount to almost half of the total losses, their omission from the analysis makes any model, even if it is a model that approximates the liquid compressibility and gas behaviour as precisely as possible, very imperfect. A simple model such as the one considered here then seems to be more than adequate, since all that one can often deduce about bubble dynamics are trends only. In other words, improvements of a few percentages are considered to be insignificant in view of the overall uncertainty of about 50%.

For all that, in order not to increase the error excessively, the curves displayed in Figures 2-12 were computed only for the limited range of parameters p_∞ and P^*_{m1}. At *higher pressures* p_∞ the limitations follow from equations (3) and (4) which are valid for ideal gases only. Real gases behave as ideal ones for pressures lower than 10^8–10^9 Pa [23], and hence the computed maximum pressure, P_{M1}, should not exceed this limit by too much.

At *lower pressures* p_∞ the first restriction is that $p_\infty > P_v$. (If $p_\infty < P_v$, the bubble grows due to evaporation of the liquid into the bubble.) For gas bubbles the second restriction is that $P_{m1} > P_v$. Unless this holds, evaporation of the liquid into the bubble changes pure gas bubbles into gas-vapour bubbles, and such mixed bubbles are not treated in this context.

In computing the curves presented in Figures 2-12 the *excitation techniques* have not been considered. However, when considering real-life situations, excitation techniques should be taken into account because they determine the actual amplitudes of bubble oscillations. With underwater explosions [14], for example, an increase in the ambient pressure will suppress the amplitude with which the generated bubbles oscillate. On the other hand, with the shock tube method [17], an increase of p_∞ implies that at the same time the amplitude of the bubble oscillations is also increased.

An interesting feature found for vapour bubbles is that at a certain pressure p_∞ there is a minimum of bubble oscillation intensity (see Figure 12). This finding may be important with respect to *cavitation erosion* and therefore it would deserve a more detailed investigation with use of a bubble model, taking into account the influence of a solid boundary. Fortunately, the minimum seems to occur near the pressure p_∞ under which the hydraulic equipment is often operated [5-8].

A general conclusion which can be drawn from Figures 4 and 9 is that very high pressures P_{M1} can be generated by increasing the ambient pressure p_∞. (Note that $P_{M1} = P^*_{M1} p_\infty$; thus P_{M1} increases even if P^*_{M1} decreases with p_∞.) This also may be an important factor when considering the damage the oscillating bubbles cause to nearby objects.

Note also that, for vapour bubbles, there are always two values of the pressure p_∞ to be found for which the amplitudes A_1 and the maximum temperatures of the vapour in the bubble are almost the same. However, the maximum pressures P_{M1} and the radiated acoustic energy differ substantially in the two cases.

5. CONCLUSION

In the paper theoretical data have been presented to demonstrate the influence of the ambient pressure on the behaviour of gas and vapour bubbles. Unfortunately, the experimental data published in the literature are rather scarce so that it has not been possible at present to check the validity of the theory.

It was found that by increasing the ambient pressure p_∞, very high maximum pressures P_{M1} and damping factors α_1 result. Another interesting observation is that there is a pressure p_∞ for which the vapour bubbles oscillate with minimum intensity. This finding may be important with respect to cavitation erosion.

The results presented can be used in evaluating the experimental records published in references [5-17] (the author intends to present examples of such evaluations elsewhere) and in predicting the bubble behaviour for different environments.

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APPENDIX: NOMENCLATURE

In this list of nomenclature the first and second symbols denote dimensional and non-dimensional quantities, respectively. The numbers in subscripts (omitted in the nomenclature but used in the text) denote the oscillation period.

t, t_z	time
R, Z	bubble radius
R_M, Z_M	maximum bubble radius
R_m, Z_m	minimum bubble radius
R_e, Z_e	equilibrium bubble radius
R_{vg}, Z_{vg}	transition radius between ideal vapour and ideal gas bubble behaviour
$\dot{R}$, $\dot{Z}$	bubble wall velocity
$\dot{R}_{vg}$, $\dot{Z}_{vg}$	transition velocity between ideal vapour and ideal gas bubble behaviour
$\ddot{R}$, $\ddot{Z}$	bubble wall acceleration
P, P^*	pressure in the liquid at the bubble wall
P_M, P_M^*	maximum pressure
P_m, P_m^*	minimum pressure
P_v, P_v^*	liquid vapour pressure
p_p, p_{zp}	peak pressure in the bubble pulse
T_c, T_{zc}	collapse or compression time of the bubble
ΔT_v, ΔT_{zv}	interval during which the bubble behaves like an ideal vapour bubble
ΔT_g, ΔT_{zg}	interval during which the bubble behaves like an ideal gas bubble
c_∞, c_∞^*	speed of sound in the liquid
A	amplitude of oscillation
p_∞	ambient pressure in the liquid
ρ_∞	liquid density
r	point in the liquid
γ	polytropic exponent of the gas or vapour in the bubble
α	damping factor