

OSCILLATIONS OF GAS BUBBLES GENERATED BY UNDERWATER EXPLOSIONS

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1. INTRODUCTION

Examination of data obtained in experiments with gas bubbles generated by underwater explosions carried out in previous study [1] has indicated several interesting facts regarding the bubble behaviour. The questions raised concern, for example, the value of the bubble oscillations amplitude, the amount and the nature of energy losses, and the extent of validity of the bubble model.

As one can assume that other bubbles, such as cavitation, spark, and laser generated bubbles behave in much the same manner it was deemed worthwhile to examine the problem in greater depth. For this purpose the data published by Arons et al. [2] and Arons and Yennie [3] that have already been examined in reference [1] are supplemented by some additional facts from works [4, 5] and an improved bubble model is used to interpret the experimental data. It is hoped that in this way a more complete picture of the phenomena will be obtained.

For the problem under examination the selection of a suitable bubble model is in many respects crucial, because the processes associated with the early stages of explosions, such as detonation and shock waves, are highly complex. Yet, in spite of great theoretical and experimental efforts (see, e.g. references [6–15]), not all the aspects of the phenomena have been fully elucidated to date. In addition, the theoretical computations are usually based on a finite difference form of the governing differential equations [6–10], which means that large amounts of computing time are required to solve them.

As the emphasis in this work will be on the later stages of bubble oscillations the model can be substantially simplified and hence large savings in computing time will be achieved. However, as a consequence of this simplification the detonation processes and the shock waves cannot be adequately dealt with.

The layout of the present work is as follows: in the next section a suitable bubble model will be specified. The results obtained with this model will be compared with experimental data in Section 3. Finally, in Section 4 energy partition in underwater explosions will be discussed.

2. PROBLEM FORMULATION

Let a spherical charge of a radius R_{m0} detonate in water. The exothermal reaction in the explosive will produce gases of very high pressure and temperature. When the detonation wave reaches the charge-liquid interface a shock wave radiates into the water and the products of explosion (gas bubble) start expanding. During the expansion phase the bubble grows to a maximum radius, R_{M1} , and then performs several damped oscillations around its equilibrium radius, R_e .

For further work it is convenient to introduce a system of non-dimensional variables. In this paper we shall use the expansion W system of variables, in which the non-dimensional time, bubble radius, energy, pressure at the bubble wall, and pressure in the liquid at a point r are defined as [16]

$$t_w = t/[R_{m0}(\rho_\infty/p_\infty)^{1/2}], \quad W = R/R_{m0}, \quad E_w = E/E_{pm0}, \\ P^* = P/p_\infty, \quad p_w = [(p/p_\infty) - 1](r/R_{m0}).$$

Here ρ_∞ is the density of the undisturbed liquid, p_∞ the ambient pressure in the liquid, and $E_{pm0} = \frac{4}{3}\pi\rho_\infty R_{m0}^3$ the minimum potential energy of the liquid.

The bubbles generated by explosions belong to a class of gas bubbles which are characterized by the fact that excitation for free oscillations is achieved by increasing their energy [17]. A general analysis of such bubbles was given in [16] where the computations of the bubble wall motion were performed with Gilmore's equation (see Appendix A for the definition of Gilmore's model) and on the assumption of an ideal gas in the bubble interior that undergoes adiabatic changes and of a uniform pressure field. In such a case the pressure at the bubble wall varies as [16]

$$(1) \quad P^* = P_{M0}^* W^{-3\gamma},$$

where P_{M0}^* is the initial maximum gas pressure at a time $t_w = 0$ and γ is the ratio of specific heats.

As mentioned above, one of the objectives of this work is to compare an improved bubble model with experiments. The energy relations necessary for this purpose are summarized in Appendix B, and experimental data on bubble oscillations given in [2-4] are summarized and transformed into the non-dimensional form in Appendix C.

Let us first examine how far the simple model discussed in [16] is appropriate for explosion generated bubbles. For example, the first maximum bubble wall radius as determined in experiments is given by equation (C3). For shallow explosions or laboratory tank experiments we may put $p_\infty = 100$ kPa and thus obtain from equation (C3) that $W_{M1} = 29.4$. This value can be directly confronted with the scaling functions $W_{M1} = W_{M1}(P_{M0}^*, \gamma)$ given in [16]. It will be seen that for $\gamma = 1.25$ [4] and $W_{M1} = 29.4$ the corresponding initial pressure equals $P_{M0} = P_{M0}^* p_\infty = 1.2$ GPa.

The first expansion time T_{we1} can be evaluated in the same manner. From equation

(C5) it follows for $p_\infty = 100$ kPa that $T_{we1} = 29.2 (T_{wo1} \doteq T_{wo1}/2, T_{wo1}$ is the period of the first bubble oscillation), and from the graph $T_{we1} = T_{we1}(P_{M0}^*, \gamma)$ it can be seen that the corresponding initial pressure is $P_{M0} = P_{M0}^* p_\infty = 1.33$ GPa ($\gamma = 1.25$).

Finally, the energy released by the explosion is given by equation (C12). For $p_\infty = 100$ kPa one obtains that $E_{we0} = 67\ 200$. Assuming that this energy equals the initial internal energy of gaseous products, E_{wi0} , it follows from equation (B2) that for $\gamma = 1.25$ the initial pressure is $P_{M0} = P_{M0}^* p_\infty = 1.6$ GPa.

All these initial pressures are much lower than the pressures usually considered for TNT [4, 5]. This finding, however, is not surprising. As said above, the model used in [16] assumed an ideal gas and a uniform pressure field in the bubble interior, which is obviously not true during the early stages of the explosion. For example, when the detonation wave reaches the interface, the gas has a density equal to that of the original solid explosive and hence can hardly be treated as an ideal gas. Similarly, due to a complex wave system the bubble interior is far from being homogeneous at that time [6].

To obtain better results a more realistic equation of state for the gas is necessary. In this work we shall use the two-gamma approximation suggested by Epstein [18]. In this case the pressure equation (1) is replaced by the relation

$$(2) \quad P^* = P_{M0}^{*'} W^{-3\gamma'} + P_{M0}^{*''} W^{-3\gamma''},$$

and at a first approximation the internal energy of the gas can be written as

$$(3) \quad E_{wi} = \frac{P_{M0}^{*'}}{\gamma' - 1} W^{-3(\gamma' - 1)} + \frac{P_{M0}^{*''}}{\gamma'' - 1} W^{-3(\gamma'' - 1)}.$$

Here $\gamma' = 3$ and $\gamma'' = 1.25$ [18].

If $W = W_{m0} = 1$ we obtain from equations (2) and (3) that

$$(4) \quad P_{M0}^* = P_{M0}^{*'} + P_{M0}^{*''},$$

and

$$(5) \quad E_{wi0} = \frac{P_{M0}^{*'}}{\gamma' - 1} + \frac{P_{M0}^{*''}}{\gamma'' - 1}.$$

For given P_{M0}^* and E_{wi0} , the equations (4) and (5) can be solved to yield $P_{M0}^{*'}$ and $P_{M0}^{*''}$. A further obvious requirement on the parameters $P_{M0}^{*'}$ and $P_{M0}^{*''}$ is that the quantities computed from the equation of motion fit the experimental data with reasonable accuracy.

Some of the experimental data suitable for comparison have been mentioned earlier (W_{M1} , T_{we1} , and E_{wi0}). Another quantity of interest is the amplitude A_1 , which was determined in [1] both from the peak pressure p_{p1} ($A_1 = 2.05$) and from the time of the first bubble oscillation T_{o1} ($A_1 = 2.02$). These two amplitudes correspond to the ambient pressure $p_\infty = 1.62$ MPa. The quantities p_{p1} and T_{o1} can

also be used in this context. Their values in the expansion system and for $p_\infty = 1.62$ MPa are (equations (C5) and (C8)) $T_{w01} = 23.0$ and $p_{wp1} = 87.4$. Finally, a quantity that can also be used is the acoustical energy associated with the first bubble pulse. This energy equals (equation (C10)) $E_{wbp} = 502$.

3. RESULTS OF COMPUTATIONS

To fit all the experimental data mentioned as best as possible a number of trial computations were carried out with Gilmore's model for different values of $P_{M0}^{*'}$ and $P_{M0}^{*''}$. In selecting the value of these parameters two approaches were adopted:

- (i) The initial internal energy was assumed to be $E_{wi0} = 67\,200$ ($p_\infty = 100$ kPa) and for a selected value of P_{M0}^* the pressures $P_{M0}^{*'}$ and $P_{M0}^{*''}$ were determined from equations (4) and (5).
- (ii) The selection of the parameters P_{M0}^* , $P_{M0}^{*'}$, and $P_{M0}^{*''}$ was partially guided by the equation of state data for detonation products computed by Kuznetsov and Shvedov [19] (for $\rho_e = 1.6 \times 10^3$ kg m⁻³).

The initial pressures and energies determined in this way are given in Table I. Using the values from Table I the equation of motion (A1) was solved for ambient pressures $p_\infty = 100$ kPa and $p_\infty = 1.62$ MPa. With regard to available experimental data the computations were terminated at W_{M1} in the first case and at W_{M2} in the second case. The results of computations and the corresponding experimental data are summarized in Tables II and III.

Table I. Values of the initial pressures and energies used in computations. The values of the energy E_{wi0} correspond to the ambient pressure $p_\infty = 100$ kPa.

P_{M0} [GPa]	6	12
$P_{M0}^{*'}$ [GPa]	4.937	10.75
$P_{M0}^{*''}$ [GPa]	1.063	1.25
E_{wi0} [-]	67 200	103 750

Table II. Comparison of theoretical and experimental data determined for the ambient pressure $p_\infty = 100$ kPa.

	Theoretical values		Experimental data
	$P_{M0} = 6$ GPa	$P_{M0} = 12$ GPa	
W_{M1}	29.29	30.87	29.35
A_1	2.47	2.49	—

Table III. Comparison of theoretical and experimental data determined for the ambient pressure $p_\infty = 1.62$ MPa.

	Theoretical values		Experimental data
	$P_{M0} = 6$ GPa	$P_{M0} = 12$ GPa	
W_{M1}	11.06	11.67	11.62
A_1	1.96	1.98	2.02—2.05
W_{m1}	2.13	2.20	—
T_{wo1}	21.97	23.10	23.0
p_{wp1}	87.1	98.2	87.4
W_{M2}	9.46	9.93	7.9

From Tables I, II, and III several important conclusions can be drawn. First, it can be seen that there is relatively strong agreement between theory and experiment up to the first minimum radius W_{m1} . However, there is a large difference as far as the value of W_{M2} is concerned. This difference is due to the energy losses that were not taken into account in the theoretical model. An interesting result is that the value of p_{wp1} (and hence also of W_{m1}) does not seem to be substantially affected by these losses. This would indicate that the kinetic energy of the liquid was almost fully transferred into the internal energy of the gas. However, it does not seem as if all the internal energy of the gas was transferred during the expansion phase back into the kinetic or even later into the potential energy of the liquid [20].

Second, though during explosions large amounts of energy are released and high initial pressures generated, the bubbles are excited to oscillate only with moderate amplitudes ($A_1 < 2.5$). Hence, with the exception of the early phases relatively simple bubble models can be used to obtain satisfactory results. Also, the predictions given in [16] regarding the maximum amplitude, A_1 , that can be achieved in the expansion system, are thus confirmed.

Third, the bubble behaviour at the later stages seems to be influenced only little by the initial conditions. For example, a comparison of the data in Tables II and III shows that doubling the initial pressure P_{M0} has little influence on the later bubble histories. This is a consequence of the fact that an increase in the initial energy is almost fully compensated for (from the point of view of the bubble oscillations) by an increase in energy radiated in the shock wave.

4. ENERGY PARTITION

Let us now examine the energy partition in underwater explosions. The results presented here are based on computations with the initial pressure $P_{M0} = 6$ GPa and ambient pressure $p_\infty = 1.62$ MPa. The respective energies can be found by means

Table IV. Energy partition at the time of the first and second maximum bubble radius.
 $p_\infty = 1.62$ MPa.

W	E_w	Results based on computed radii		Results based on measured radii		Energy partition determined in reference [9]	
		[—]	[%]	[—]	[%]	[cal g ⁻¹]	[%]
$W = W_{m0}$	E_{we0}	4148	100	4148	100	1050	100
$W = W_{M1}$	E_{wi}	433	10.5	417	10	95	9
	ΔE_{wpp}	1350	32.5	1568	37.8	385	36.7
	E_{wsh}	2365	57	2163	52.2	570	54.3
$W = W_{M2}$	E_{wi}	486	12	557	13.4	60	5.7
	ΔE_{wpp}	847	20	492	11.8	120	11.4
	ΔE_{wa}	2815	68	2665	64.3	690	65.8
	E_{wbp}	450	11	502	12.1	120	11.4
	ΔE_{wdu}	0	0	434	10.5	180	17.1

of the data given in Tables I and III and the equations (3), (5), (B5)–(B9), and (C11).

The energy partition data determined in this way are summarized in Table IV. The first column corresponds to energies calculated using the theoretical radii W_{M1} and W_{M2} (these quantities are given in the first column in Table III). The second column in Table IV is based on measured values of W_{M1} and W_{M2} (the third column in Table III). For comparison, the third column in Table IV contains energy partition data found by Arons and Yennie [3] and Arons [21]. For easier identification in the source works these data have been left in original units (cal g⁻¹).

Though Arons and Yennie [3] and Arons [21] used a different approach, their energy partition for $W = W_{M1}$ is very close to the results arrived at in this work (the results based on experimental data). Let us only note that due to a slight difference in the values of Q_e and Q their initial energy is not exactly the same as that given here (Arons and Yennie [3] use normalized energies $E_g = E/G$, which are related to the expansion variables by a relation $E_w = E_g Q_e/p_\infty$; here G is the weight of the charge).

A more pronounced difference in energies determined for $W = W_{M2}$ should be attributed to the ways the internal energies are calculated in the two works. The method of Arons and Yennie is based on the formula

$$(6) \quad E_{T2} = E_{T1}(T_{o2}/T_{o1})^3,$$

in which E_{T1} and E_{T2} are the total energies ($E_T = E_{pM} + E_{im}$, E_{pM} is the maximum potential energy and E_{im} minimum internal energy, see Appendix B) associated with the first and second oscillations, and T_{o1} , T_{o2} the times of the first and second oscillations, respectively.

Equation (6) represents a simplified version of the exact formula [1]

$$(7) \quad E_{pM2} = E_{pM1}(R_{M2}/R_{M1})^3.$$

If the ratio of the maximum radii is substituted by the ratio of the oscillation times, a minor error $\sim 0.98^3$ is introduced [1]. However, if the potential energies are substituted by the total energies, quite erroneous results can ensue, because the internal energies are of the same order as the potential energies (cf. Table IV).

It is difficult to give a source for the 10.5% dissipated unaccounted energy. In the literature various causes are suggested: (i) turbulence induced in the water surrounding the bubble [3, 4], (ii) loss of the gas from the main bubble in the form of micro-bubbles [3, 22], (iii) gas cooling in the protuberances [23], and (iv) internal converging shocks [24]. Evidently, further experiments are needed to solve this question.

5. CONCLUSION

Oscillations of gas bubbles generated by underwater explosions were examined in detail. It was found that the theoretical model agrees with experiments in an interval $(0, T_{o1})$. For times $t > T_{o1}$ large energy losses, not accounted for by the model, occur. The nature of these losses has not been clarified yet. Among possible candidates are turbulence, heat and gas losses from the bubble, and converging shocks in the bubble interior.

The early stages of explosions are highly complex and no attempt was undertaken to analyze them. Here the simple two-gamma model was used to bridge this phase. However, it should be emphasized that any detailed physical interpretation of the quantities P_{M0} and γ is tenuous at best and they should be regarded rather as parameters affording a useful fit of simplified theory to experimental results [3].

Examination of the experiments [3, 4] reveals two important facts regarding the bubbles oscillating with amplitudes $A_1 < 2.5$. First, though the bubble walls may exhibit large deformations and sharp irregularities (protuberances) at the first minimum volume and at later times, the bubbles are stable and do not break up. Second, the pressure vs time records of the bubble pulses never contain jumps that could be ascribed to the shock front.

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APPENDIX A.
GILMORE'S MODEL

In Gilmore's model the equation of motion for the bubble wall has the form (for the dimensional form see, e.g. references [1, 25])

$$(A1) \quad \ddot{W}W(1 - \dot{W}/C^*) + \frac{3}{2}\dot{W}^2(1 - \frac{1}{3}\dot{W}/C^*) = H^*(1 + \dot{W}/C^*) + (W/C^*)\dot{H}^*(1 - \dot{W}/C^*),$$

where the velocity of sound in the liquid at the wall, C^* , equals

$$(A2) \quad C^* = c_\infty^*[(P^* + B^*)/(1 + B^*)]^{(n-1)/2n},$$

and the enthalpy change between pressures P and p_∞ is given by

$$(A3) \quad H^* = (n-1)^{-1}n(1+B^*)\{[(P^*+B^*)/(1+B^*)]^{(n-1)/n} - 1\}.$$

Here $c_\infty^* = c_\infty(\rho_\infty/p_\infty)^{1/2}$ is the velocity of sound in the undisturbed liquid, and $B^* = B/p_\infty$, n are constants in the Tait equation of state for the liquid. For water, which we consider here, $c_\infty = 1450 \text{ m s}^{-1}$, $B = 300 \text{ MPa}$, $n = 7$, and $\rho_\infty = 10^3 \text{ kg m}^{-3}$. The dots in equation (A1) denote differentiation with respect to time.

The peak pressure in the bubble pulse can be conveniently computed from a simple formula [20]

$$(A4) \quad p_{wp1} = W_{m1}(P_{M1}^* - 1).$$

APPENDIX B.
ENERGIES IN THE EXPANSION SYSTEM

The energy relation in the expansion system can be written as [17, 20]

$$(B1) \quad \Delta E_{wi} = E_{wk} + \Delta E_{wp} + \Delta E_{wd},$$

where the initial internal energy of the ideal gas (referred to infinite adiabatic expansion) equals

$$(B2) \quad E_{wi0} = P_{M0}^*/(\gamma - 1).$$

For a wall position W the internal energy is

$$(B3) \quad E_{wi} = \frac{P_{M0}^*}{\gamma - 1} W^{-3(\gamma-1)},$$

and hence the change in the internal energy can be written as

$$(B4) \quad \Delta E_{wi} = E_{wi0} - E_{wi} = \frac{P_{M0}^*}{\gamma - 1} [1 - W^{-3(\gamma-1)}].$$

The change in the potential energy of the liquid is given by

$$(B5) \quad \Delta E_{wp} = W^3 - 1.$$

Finally E_{wk} and ΔE_{wd} designate the kinetic and dissipated energy, respectively. When $W = W_m$ and $W = W_M$ it can be assumed that the kinetic energy $E_{wk} \doteq 0$.

Energy associated with a shock wave can be determined from a relation

$$(B6) \quad E_{wsh} = \Delta E_{wi} - \Delta E_{wp},$$

where ΔE_{wi} and ΔE_{wp} are evaluated for $W = W_{M1}$. Similarly, theoretical energy associated with the first bubble pulse is

$$(B7) \quad E_{wbp} = \Delta E_{wi} - \Delta E_{wp} - E_{wsh}$$

where ΔE_{wi} and ΔE_{wp} are evaluated for $W = W_{M2}$ and E_{wsh} is given by equation (B6). Then the total acoustical energy radiated up to W_{M2} is

$$(B8) \quad \Delta E_{wa} = E_{wsh} + E_{wbp}.$$

In the case of experimental data the energy of the first bubble pulse is given by equation (C10). Then the unaccounted energy dissipated between W_{M1} and W_{M2} equals

$$(B9) \quad \Delta E_{wdu} = \Delta E_{wi} - \Delta E_{wp} - \Delta E_{wa},$$

and the energies are evaluated from experimental data now.

APPENDIX C.

EXPERIMENTAL DATA [1-4]

In research of underwater explosions with TNT charges it was established that the first maximum bubble radius, R_{M1} , the weight of the charge, G , and the ambient pressure, p_∞ , are related by a formula

$$(C1) \quad R_{M1} = 72.4 (G/p_\infty)^{1/3} \quad [\text{m, kg, Pa}].$$

The weight of the spherical charge equals

$$(C2) \quad G = \frac{4}{3}\pi R_{m0}^3 \rho_e \quad [\text{kg, m, kg m}^{-3}],$$

where ρ_e is the density of the explosive. In the case of TNT the density is $\rho_e = 1.6 \times 10^3 \text{ kg m}^{-3}$ [5]. Then upon substituting equation (C2) into relation (C1) we obtain

$$(C3) \quad W_{M1} = R_{M1}/R_{m0} = 1366/(p_\infty)^{1/3}.$$

It was also found that the period of the first bubble oscillation is

$$(C4) \quad T_{o1} = 4576 G^{1/3}/(p_\infty)^{5/6} \quad [\text{s, kg, Pa}].$$

Again, using equation (C2) we obtain for water that

$$(C5) \quad T_{wo1} = T_{o1} / [R_{m0} (Q_{\infty} / p_{\infty})^{1/2}] = 2710 / p_{\infty}^{1/3} .$$

The formulae (C1) and (C4) were verified for a large extent of ambient pressures. The measured peak pressure in the first bubble pulse was

$$(C6) \quad p_{p1} = 8.27 \times 10^6 \text{ [Pa]} ,$$

where the measurement was performed at a point

$$(C7) \quad r = 1.13 G^{1/3} \text{ [m, kg]} ,$$

and the ambient pressure was $p_{\infty} = 1.62 \text{ MPa}$. Then using equation (C2) the non-dimensional peak pressure equals

$$(C8) \quad p_{wp1} = (p_{p1} / p_{\infty} - 1) r / R_{m0} = 87.4 .$$

Finally, the acoustical energy flux in the first bubble pulse determined at the point r and for $p_{\infty} = 1.62 \text{ MPa}$ was

$$(C9) \quad F_1 = 31.7 \times 10^3 G^{1/3} \text{ [kg s}^{-2}, \text{ kg]} .$$

Hence the energy carried away by the first bubble pulse equals ($E_{pm0} = \frac{4}{3} \pi p_{\infty} R_{m0}^3$)

$$(C10) \quad E_{wbp} = 4\pi r^2 F_1 / E_{pm0} = 502 .$$

The last experimental datum that can be used here concerns the energy released by the explosion. In non-dimensional form this energy equals

$$(C11) \quad E_{we0} = Q G / E_{pm0} = Q Q_e / p_{\infty} ,$$

where Q is the detonation energy per 1 kg of the explosive. In the case of TNT the detonation energy is $Q = 4.2 \text{ MJ kg}^{-1}$ [5]. Thus one obtains

$$(C12) \quad E_{we0} = 6.72 \times 10^9 / p_{\infty} .$$

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A simple yet reasonably accurate model of gas bubbles generated by underwater explosions is presented. The results obtained with this model are compared with experimental data. Finally the energy partition problem in underwater explosion phenomena is examined. It is shown that the theoretical model agrees with experimental data up to the first bubble minimum volume. At later stages unaccounted energy losses cause a large deviation of the model from experiments. It is assumed that the conclusions drawn here can be applied to other kinds of bubbles, such as cavitation, spark, and laser generated bubbles.

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