APPLICATION OF THE GROUP PULSE PROCESSES IN THE THEORY OF BARKHAUSEN NOISE

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In the paper the 2nd-order periodic group pulse process is considered a statistical model of the Barkhausen voltage. Using this model an analytical formula for the power spectrum is derived and further specified to fit the measured spectrograms of Barkhausen noise. The results are interpreted with respect to the generating physical mechanism and examples of several numerical values are given.

1. INTRODUCTION

Let us consider a ferromagnetic specimen suitably placed in the vicinity of a pick-up coil, e.g. in its centre. If a variable external magnetic field is applied to the specimen (this field may be produced either by the passage of a variable current through a driving coil winding or by the movement of a permanent magnet in the specimen vicinity), voltage $u(t)$ will be induced in the pick-up coil, in which two components may be distinguished [1]:

$$u(t) = u_D(t) + u_C(t) . \tag{1}$$

The first component, $u_D(t)$, depends deterministically (generally in a non-linear way) on the applied field. If the applied field is periodical, the component $u_D(t)$ will be represented by discrete lines in the spectrum of voltage $u(t)$. If a ferromagnetic material is used as a transformer core, the voltage $u_D(t)$ will be a useful voltage (signal).

The second component, $u_C(t)$, is random, has a continuous power spectrum and in the case of the mentioned transformer it represents noise that is added to the useful voltage $u_D(t)$.

This division of voltage $u(t)$ into its components was carried out from the point of view of the statistical dependence between an applied field and voltage $u(t)$. Another possible division of voltage $u(t)$ into components takes into account the generating physical mechanism. If thermal noise $u_T(t)$ and voltage $u_A(t)$ induced in the pick-up coil directly by the change of the applied field are neglected (these components are usually much smaller compared with the components to be regarded further), then voltage $u(t)$ may be divided into two components again, namely

$$u(t) = u_I(t) + u_R(t) . \tag{2}$$

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The first component, $u_I(t)$, is generated by the irreversible jumps of the domain walls known as the Barkhausen jumps. The second component, $u_R(t)$, is generated by the reversible domain wall motions and by other reversible magnetizing processes in the ferromagnetic specimen.

Both the irreversible component $u_I(t)$ and the reversible component $u_R(t)$ contribute both to the voltage $u_D(t)$ and to the voltage $u_C(t)$, so that we may write

$$u_I(t) = u_{ID}(t) + u_{IC}(t), \tag{3}$$

$$u_R(t) = u_{RD}(t) + u_{RC}(t), \tag{4}$$

or

$$u_D(t) = u_{ID}(t) + u_{RD}(t), \tag{5}$$

$$u_C(t) = u_{IC}(t) + u_{RC}(t). \tag{6}$$

Equations (1)–(6) will be used to define several terms. The voltage $u_I(t)$, which is generated by the Barkhausen jumps, will be referred to as the Barkhausen voltage. According to equation (3) this voltage has two components, the deterministic one $u_{ID}(t)$ and the random one $u_{IC}(t)$. The random component $u_{IC}(t)$ will be referred to, in accordance with the established tradition, as Barkhausen noise [1, 2]. Let us remark that it is not possible to separate the two components $u_I(t)$ and $u_R(t)$ one from the other easily (if ever) during the measurement. Thus in experiments it is the component $u_C(t)$ that is analysed as "Barkhausen noise" and not the $u_{IC}(t)$.

In this paper we shall be concerned with the Barkhausen voltage $u_I(t)$ only. During an irreversible domain wall jump a voltage pulse is induced in the pick-up coil. As Barkhausen jumps occur at random in time and space (within the specimen volume) and as volume participating in a jump is also random, the Barkhausen voltage is formed of a random time sequence of random voltage pulses. Therefore pulse random processes are exclusively used as Barkhausen noise statistical models. The simplest model is based upon the homogeneous Poisson pulse process (the shot noise model) [3]. Due to its simplicity this model is easy to work with; however, it approximates Barkhausen noise only very roughly because it does not consider the observed pulse clustering [4–6]. This pulse clustering was first taken into account by Mazzetti and Montalenti [7], who approximated Barkhausen noise by an aperiodic pulse process with an attractive correlation between pulses[1]). In this paper the 2nd-order periodic group pulse process will be used as a Barkhausen voltage model[2]). This process represents further generalization of the 1st-order group pulse processes studied in papers [8] and [9].

[1]) This and some other terms from the theory of random pulse processes are defined in papers [8, 9].

[2]) The term "the order of the group pulse process" is used here for the first time and therefore it deserves some explanation. As the 0th-order pulse process we denote such a process at which no systematic pulse clustering occurs. If the pulse groups occur, we shall speak about the 1st-order group pulse process and if the group clusters occur, the process will be of the 2nd-order.

Understanding the Barkhausen voltage may be useful for several reasons. First, the Barkhausen pulses represent a unique means for a physicist studying the processes inside the ferromagnetic material and as such they have been used in basic physical research for a long time [4, 10, 11]. However, this feature of the Barkhausen voltage seems to be equally useful in applications for nondestructive testing of materials [12–14]. Finally, Barkhausen noise may be a limiting factor in increasing the sensitivity of ferromagnetic devices and here good understanding of the processes involved is also necessary [15].

2. STATISTICAL MODEL OF THE BARKHAUSEN VOLTAGE

Let us apply a periodic magnetic field $H(t)$ to a ferromagnetic specimen, so that the specimen is periodically taken through the hysteresis loop. As was said above, during irreversible jumps of the domain walls voltage pulses are induced in the pick-up coil. These pulses may occur isolately; however, they occur more frequently in groups. The mean density of groups and the mean number of pulses in groups are not uniform during the magnetization cycle; the greatest groups and the greatest density of the groups occur in the steepest part of the hysteresis loop, that is in the surroundings of the coercitive field $\pm H_c$. When the direction of magnetization is reversed the polarity of induced pulses changes simultaneously. This brief description was freely adapted from the book by Bittel and Storm [1] and the physical processes described above are schematically shown in fig. 1.

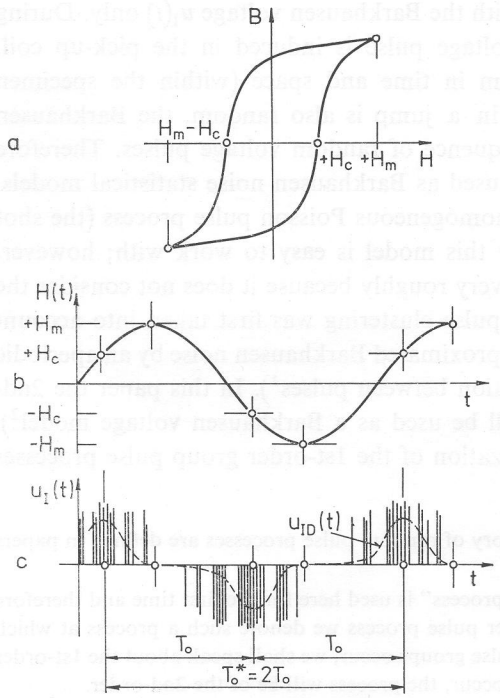

Fig. 1. Barkhausen voltage induced in a pick-up coil when a specimen is periodically taken through the hysteresis loop (schematically). a) Hysteresis loop. b) Time varying applied field. c) Barkhausen voltage.

As indicated in fig. 1, the clusters of pulse groups occur in the neighbourhood of the points $\pm H_c$ and because of the periodicity of the function $H(t)$ these clusters will also occur "periodically" along the time axis. The polarity of pulses in a certain cluster is just opposite to that in the preceding one or in the following one. The course of the component $u_{ID}(t)$ is plotted by the dashed line in fig. 1.

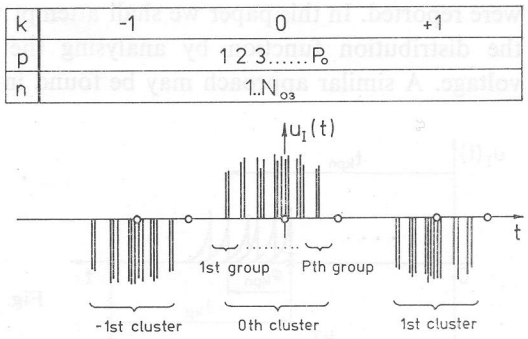

Fig. 2. Numbering of pulses, groups and clusters of Barkhausen voltage.

To be able to describe this sequence of voltage pulses mathematically we shall introduce the following terms: the pulse reference point, the group reference point and the cluster reference point. These will be those points that will unambiguously define the position of pulses, groups and group clusters with respect to the time axis origin. As the pulse reference point we shall use an instant when the pulse reaches its maximum. The group reference point will be identified with a reference point of the first pulse in the group. Finally, the instant when an applied field equals $\pm H_c$ will be used as the reference point of the group cluster.

Let us identify the time axis origin with a certain group cluster reference point and let us denote the clusters by the subscript k where $k = 0, \pm 1, \ldots$, the groups in clusters by the subscript p where $p = 1, \ldots, P$, and the pulses in groups by the subscript n where $n = 1, \ldots, N$ (fig. 2). Thus any cluster is determined by one subscript (k), any group by two subscripts (kp) and any pulse by three subscripts (kpn). The number of groups in the kth cluster is then P_k and the number of pulses in the pth group of the kth cluster is N_{kp}.

The form of each pulse may be approximated analytically by a certain time function $f_{kpn}(t)$, which may in general differ for each pulse.

Let us denote by φ_{kpn} the distance of the nth pulse in the pth group of the kth cluster from the reference point of the pth group in the kth cluster and by λ_{kp} the distance of the reference point of the pth group in the kth cluster from the reference point of the kth cluster. Then the position t'_{kpn} of the kpnth pulse with respect to the time axis origin equals (fig. 3)

$$t'_{kpn} = kT_0 + \lambda_{kp} + \varphi_{kpn} . \tag{7}$$

The functions $f_{kpn}(t)$ and the variables φ_{kpn}, λ_{kp}, N_{kp} and P_k are random and thus

their complete statistical description is given by their distribution function. From the statistical point of view this distribution function represents the maximum information we may obtain about the Barkhausen voltage and it is therefore desirable to obtain its best possible estimate. This estimate has not been determined experimentally in full extent yet; however, in some works partial results, as the distribution of time intervals between pulses [5] and the distribution of pulse heights [4], were reported. In this paper we shall attempt to obtain some information regarding the distribution function by analysing the power spectrum of the Barkhausen voltage. A similar approach may be found in papers [1–3, 7, 15–19].

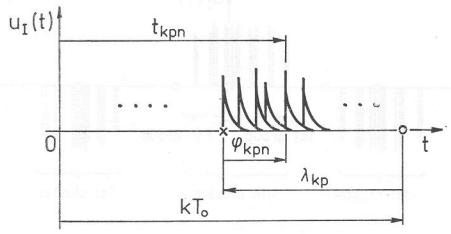

Fig. 3. Determination of a pulse position with respect to the origin.

For the purpose of this analysis a statistical model of the Barkhausen voltage will be postulated first. To make the discussion clearer we shall distinguish between the physical reality and a mathematical model by using different terms. When speaking about the Barkhausen voltage/noise we mean the respective part of the voltage $u(t)$ obtained in an experiment. On the contrary, the statistical model of this voltage/noise will be called the random process and denoted as $\xi(t)$.

To create the statistical model we shall suppose that the form of all pulses may be approximated by the same time function $f(t, \boldsymbol{a})$, where $\boldsymbol{a}$ is an r-dimensional random vector of r pulse random parameters as amplitude, time constant etc. We shall suppose further that all pulses have the same polarity. Then the process $\xi(t)$ may be written in the form

$$\xi(t) = \sum_{k=-\infty}^{+\infty} \sum_{p=1}^{P_k} \sum_{n=1}^{N_{kp}} f(t - kT_0 - \lambda_{kp} - \varphi_{kpn}, \boldsymbol{a}_{kpn}) . \tag{8}$$

As may be seen from the structure of the expression (8) the process $\xi(t)$ represents the 2nd-order periodic group pulse process.

Let us now suppose that the random variables $\boldsymbol{a}_{kpn}$, φ_{kpn}, λ_{kp}, N_{kp} and P_k are mutually independent and that the single random variables are also independent for different values of subscripts k, p and n. Let us also suppose that the distribution functions of these variables do not depend on the value of the mentioned subscripts. Then the random pulse process $\xi(t)$ is fully determined by the probability densities $w_r(\boldsymbol{a})$, $w_1(\varphi)$, $w_1(\lambda)$, $w_1(N)$ and $w_1(P)$, by the function $f(t, \boldsymbol{a})$ and by the period T_0. We shall try to find an analytical expression for the power spectrum of this process in the next section.

3. DERIVATION OF THE POWER SPECTRUM FORMULA

The power spectrum $\mathscr{W}(\omega)$ of the process $\xi(t)$ will be determined from the formula

$$\mathscr{W}(\omega) = \lim_{K\to\infty} \frac{1}{(2K+1)\,T_0} \langle |S_K(\omega)| \rangle^2 , \tag{9}$$

where the symbol $\langle\ \rangle$ denotes an ensemble average and $S_K(\omega)$ is a spectrum of a realization of the process $\xi(t)$ truncated in an interval $(-T/2, +T/2)$ that fully spans $(2K+1)$ group clusters so that $T = (2K+1)\,T_0$. We shall denote this truncated realization $\xi_K(t)$ and it will hold that $\xi_K(t) = \xi(t)$ in the interval $(-T/2, +T/2)$ and $\xi_K(t) = 0$ for $|t| > T/2$. The truncated realization $\xi_K(t)$ can be written as

$$\xi_K(t) = \sum_{k=-K}^{K} \sum_{p=1}^{P_k} \sum_{n=1}^{N_{kp}} f(t - kT_0 - \lambda_{kp} - \varphi_{kpn}, \boldsymbol{a}_{kpn}) . \tag{10}$$

The spectrum $S_K(\omega)$ of this truncated realization may be obtained by the Fourier transform of the expression (10). If we denote by $s(\omega, \boldsymbol{a})$ the Fourier transform of a function $f(t, \boldsymbol{a})$, then we have

$$S_K(\omega) = \sum_{k=-K}^{K} \sum_{p=1}^{P_k} \sum_{n=1}^{N_{kp}} s(\omega, \boldsymbol{a}_{kpn}) \exp\left[-j\omega(kT_0 + \lambda_{kp} + \varphi_{kpn})\right] . \tag{11}$$

Now we shall find the modulus squared of the spectrum $S_K(\omega)$. If we denote the complex conjugate by the asterisk, we may write

$$|S_K(\omega)|^2 = S_K(\omega)\, S_K^*(\omega) = \tag{12}$$

$$= \sum_{k=-K}^{K} \sum_{p=1}^{P_k} \sum_{n=1}^{N_{kp}} \sum_{l=-K}^{K} \sum_{q=1}^{P_l} \sum_{m=1}^{N_{lq}} s(\omega, \boldsymbol{a}_{kpn})\, s^*(\omega, \boldsymbol{a}_{lqm}) \,.$$

$$. \exp\left[-j\omega(kT_0 + \lambda_{kp} + \varphi_{kpn} - lT_0 - \lambda_{lq} - \varphi_{lqm})\right] .$$

From the six-fold sum terms satisfying the following conditions can be taken out

(a) $\quad k = l\,, \quad p = q\,, \quad n = m\,,$

(b) $\quad k = l\,, \quad p = q\,, \quad n \neq m\,,$

(c) $\quad k = l\,, \quad p \neq q\,,$

(d) $\quad k \neq l\,,$

and thus the expression (12) may be rearranged to give

$$|S_K(\omega)|^2 = \sum_{k=-K}^{K} \sum_{p=1}^{P_k} \sum_{n=1}^{N_{kp}} |s(\omega, \boldsymbol{a}_{kpn})|^2 + \tag{13}$$

$$+ \sum_{k=-K}^{K} \sum_{p=1}^{P_k} \sum_{\substack{n=1 \\ n\neq m}}^{N_{kp}} \sum_{m=1}^{N_{kp}} s(\omega, \boldsymbol{a}_{kpn})\, s^*(\omega, \boldsymbol{a}_{kpm}) \exp\left[-j\omega(\varphi_{kpn} - \varphi_{kpm})\right] +$$

$$+\sum_{k=-K}^{K}\sum_{p=1}^{P_k}\sum_{\substack{n=1\\p\neq q}}^{N_{kp}}\sum_{q=1}^{P_k}\sum_{m=1}^{N_{kq}} s(\omega, \boldsymbol{a}_{kpn})\, s^*(\omega, \boldsymbol{a}_{kqm}) \,.$$

$$\cdot \exp\left[-j\omega(\lambda_{kp} + \varphi_{kpn} - \lambda_{kq} - \varphi_{kqm})\right] +$$

$$+\sum_{k=-K}^{K}\sum_{p=1}^{P_k}\sum_{\substack{n=1\\k\neq l}}^{N_{kp}}\sum_{l=-K}^{K}\sum_{q=1}^{P_l}\sum_{m=1}^{N_{lq}} s(\omega, \boldsymbol{a}_{kpn})\, s^*(\omega, \boldsymbol{a}_{lqm}) \,.$$

$$\cdot \exp\left[-j\omega(kT_0 + \lambda_{kp} + \varphi_{kpn} - lT_0 - \lambda_{lq} - \varphi_{lqm})\right] .$$

Now the expectation of (13) can be determined. With respect to the earlier postulated independence and stationarity of the random variables $\boldsymbol{a}_{kpn}$, φ_{kpn}, λ_{kp}, N_{kp} and P_k the three-fold sum in (13) will contain $(2K + 1)\langle P\rangle\langle N\rangle$ identical terms of the type

$$\langle |s(\omega, \boldsymbol{a})|^2\rangle \,. \tag{14}$$

The four-fold sum will contain $(2K + 1)\langle P\rangle\langle (N^2 - N)\rangle$ identical terms of the type

$$\langle |s(\omega, \boldsymbol{a})|\rangle^2 \, |\chi_\varphi(\omega)|^2 \,, \tag{15}$$

where $\chi_\varphi(\omega)$ is a characteristic function of the random variable φ. The five-fold sum will contain $(2K + 1)\langle (P^2 - P)\rangle\langle N\rangle^2$ identical terms of the type

$$\langle |s(\omega, \boldsymbol{a})|\rangle^2 \, |\chi_\varphi(\omega)|^2 \, |\chi_\lambda(\omega)|^2 \,, \tag{16}$$

where $\chi_\lambda(\omega)$ is a characteristic function of the random variable λ. Finally, we may take out the $\langle P\rangle^2\langle N\rangle^2$ terms of the type (16) from the six-fold sum and fill up the remaining two-fold sum so that it will be valid even for $k = l$. We obtain

$$\begin{aligned}\langle |S_K(\omega)|^2\rangle &= (2K + 1)\langle P\rangle\langle N\rangle\langle |s(\omega, \boldsymbol{a})|^2\rangle + \\ &(2K + 1)\langle P\rangle(\langle N^2\rangle - \langle N\rangle)\langle |s(\omega, \boldsymbol{a})|\rangle^2 \, |\chi_\varphi(\omega)|^2 + \\ &+ (2K + 1)(\langle P^2\rangle - \langle P\rangle)\langle N\rangle^2\langle |s(\omega, \boldsymbol{a})|\rangle^2 \, |\chi_\varphi(\omega)|^2 \, |\chi_\lambda(\omega)|^2 + \\ &+ \langle P\rangle^2\langle N\rangle^2\langle |s(\omega, \boldsymbol{a})|\rangle^2 \, |\chi_\varphi(\omega)|^2 \, |\chi_\lambda(\omega)|^2 \sum_{k=-K}^{K}\sum_{l=-K}^{K} \exp\left[-j\omega T_0(k - l)\right] - \\ &- (2K + 1)\langle P\rangle^2\langle N\rangle^2\langle |s(\omega, \boldsymbol{a})|\rangle^2 \, |\chi_\varphi(\omega)|^2 \, |\chi_\lambda(\omega)|^2 \,.\end{aligned} \tag{17}$$

Now we may substitute the expression (17) into (9) and take the limit for $K \to \infty$. Using the identity

$$\lim_{K\to\infty} \frac{1}{(2K + 1)} \sum_{k=-K}^{K}\sum_{l=-K}^{K} \exp\left[-j\omega T_0(k - l)\right] = \frac{2\pi}{T_0} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0) \tag{18}$$

the expression for the power spectrum will be obtained in the form

$$\begin{aligned}(19)\qquad \mathscr{W}(\omega) &= v_0\langle P\rangle\langle N\rangle\langle|s(\omega,\boldsymbol{a})|^2\rangle + \\ &+ v_0\langle P\rangle\left(\langle N^2\rangle - \langle N\rangle\right)\langle|s(\omega,\boldsymbol{a})|\rangle^2|\chi_\varphi(\omega)|^2 + \\ &+ v_0\left(\langle P^2\rangle - \langle P\rangle\right)\langle N\rangle^2\langle|s(\omega,\boldsymbol{a})|\rangle^2|\chi_\varphi(\omega)|^2|\chi_\lambda(\omega)|^2 - \\ &- v_0\langle P\rangle^2\langle N\rangle^2\langle|s(\omega,\boldsymbol{a})|\rangle^2|\chi_\varphi(\omega)|^2|\chi_\lambda(\omega)|^2 + \\ &+ v_0^2\langle P\rangle^2\langle N\rangle^2\langle|s(\omega,\boldsymbol{a})|\rangle^2|\chi_\varphi(\omega)|^2|\chi_\lambda(\omega)|^2\,2\pi\sum_{k=-\infty}^{\infty}\delta(\omega - k\omega_0)\,.\end{aligned}$$

Here $v_0 = 1/T_0 = f_0$, $\omega_0 = 2\pi f_0$ and $\delta(\omega)$ is the delta function.

The expression (19) is the required general formula for the power spectrum of the 2nd-order periodic group pulse process. This expression has two distinct parts: the continuous part (the first four terms on the right-hand side) and the discrete part (the last term on the right-hand side). Let us remark here that the discrete part originates in a periodical occurrence of the group clusters only.

4. SPECIFICATION OF THE POWER SPECTRUM FORMULA

In this section the distributions of the random variables P, N, φ, λ and the function $f(t, \boldsymbol{a})$ will be specified so that it will be possible to obtain from the general expression (19) a formula suitable for further work. To stress that we are dealing with the component generated by the irreversible processes the power spectrum will be denoted by the subscript I.

In applications one is usually interested in power spectrum $W(\omega)$ defined for positive circular frequencies ω only. Therefore $\mathscr{W}(\omega)$ will be replaced by $W(\omega)$, which can be easily accomplished, as

$$(20)\qquad W(\omega) = 2\mathscr{W}(\omega)\,,\quad \omega \geqq 0\,.$$

Let us suppose now that the random discrete variables P and N are Poisson distributed, so that

$$(21)\qquad \langle N^2\rangle - \langle N\rangle = \langle N\rangle^2 \quad\text{and}\quad \langle P^2\rangle - \langle P\rangle = \langle P\rangle^2\,.$$

Using relations (20) and (21) the expression (19) becomes

$$\begin{aligned}(22)\quad W_{\mathrm{I}}(\omega) &= 2v_0\langle P\rangle\langle N\rangle\langle|s(\omega,\boldsymbol{a})|^2\rangle + 2v_0\langle P\rangle\langle N\rangle^2\langle|s(\omega,\boldsymbol{a})|\rangle^2|\chi_\varphi(\omega)|^2 + \\ &+ 2v_0^2\langle P\rangle^2\langle N\rangle^2\langle|s(\omega,\boldsymbol{a})|\rangle^2|\chi_\varphi(\omega)|^2|\chi_\lambda(\omega)|^2\,2\pi\sum_{k=0}^{\infty}\delta(\omega - k\omega_0)\,.\end{aligned}$$

The form of the voltage pulses induced in a pick-up coil during a Barkhausen jump is rather complex [4]; however, here they will be to a first approximation described

by the exponential function

$$f(t, \boldsymbol{a}) \begin{cases} a \exp(-t/\theta) & t \geqq 0, \\ 0 & t < 0. \end{cases} \tag{23}$$

Here a is a random pulse amplitude and θ is a random pulse time constant. Application of the Fourier transformation to (23) gives

$$s(\omega, \boldsymbol{a}) = \frac{a\theta}{1 + j\omega\theta}. \tag{24}$$

Let us suppose that the random variables a and θ are mutually independent and that their standard deviations are small compared with the mean. Then we may write approximately

$$\langle |s(\omega, \boldsymbol{a})|^2 \rangle \doteq \langle |s(\omega, \boldsymbol{a})| \rangle^2 \doteq \frac{\langle a \rangle^2 \langle \theta \rangle^2}{1 + \omega^2 \langle \theta \rangle^2}. \tag{25}$$

To obtain an expression for the power spectrum comparable with the formula derived in paper [7] it may be supposed that the random variable φ is exponentially distributed, that is

$$w_1(\varphi) = (1/\langle \varphi \rangle) \exp(-\varphi/\langle \varphi \rangle). \tag{26}$$

In contrast to the Poisson process, where intervals ε between neighbouring pulses have exponential distribution (fig. 4a), it is the distance φ of each pulse from the group reference point that is exponentially distributed here (fig. 4b).

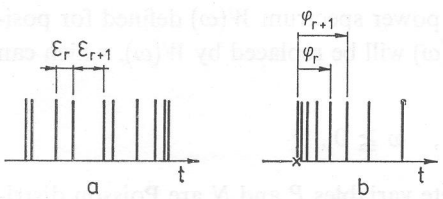

Fig. 4. Pulse spreading along time axis. a) Homogeneous Poisson process. b) Group pulse process considered in this paper.

The modulus squared of the characteristic function of the distribution (26) equals

$$|\chi_\varphi(\omega)|^2 = \frac{1}{1 + \omega^2 \langle \varphi \rangle^2}. \tag{27}$$

Finally, let us suppose that the random variable λ has the Laplace distribution (this distribution is considered because it enables simple graphical display)

$$w_1(\lambda) = \frac{1}{2\langle \lambda \rangle} \exp(-|\lambda|/\langle \lambda \rangle). \tag{28}$$

Then

$$|\chi_\lambda(\omega)|^2 = \frac{1}{(1 + \omega^2 \langle\lambda\rangle^2)^2}. \tag{29}$$

The functions (25), (27) and (29) have maxima for $\omega = 0$ and they decrease monotonously for growing ω. Let us denote as $\omega_1 = 1/\langle\lambda\rangle$, $\omega_2 = 1/\langle\varphi\rangle$ and $\omega_3 = 1/\langle\theta\rangle$. As it is always valid for the Barkhausen voltage that $\omega_3 > \omega_2 > \omega_1$, it can be written for $\omega < \omega_2$ with sufficient accuracy that

$$\langle|s(\omega, \boldsymbol{a})|\rangle^2 \doteq \langle a\rangle^2 \langle\theta\rangle^2, \qquad |\chi_\varphi(\omega)|^2 \doteq 1. \tag{30}$$

Substituting (25), (27) and (29) into (22) and using (30) we obtain ($\nu_0 = f_0 = 2f_0^*$)

$$W_I(\omega) = 2f_0\langle P\rangle \langle N\rangle \frac{\langle a\rangle^2 \langle\theta\rangle^2}{1 + \omega^2\langle\theta\rangle^2}\left[1 + \frac{\langle N\rangle}{1 + \omega^2\langle\varphi\rangle^2}\right] + \tag{31}$$

$$+ 2f_0^2\langle P\rangle^2 \langle N\rangle^2 \frac{\langle a\rangle^2 \langle\theta\rangle^2}{(1 + \omega^2\langle\lambda\rangle^2)^2} 2\pi \sum_{k=0}^{\infty} \delta(\omega - k\omega_0).$$

The first term on the right-hand side of (31) represents the continuous part $W_{IC}(\omega)$ of the spectrum, the second term represents the discrete part $W_{DC}(\omega)$.

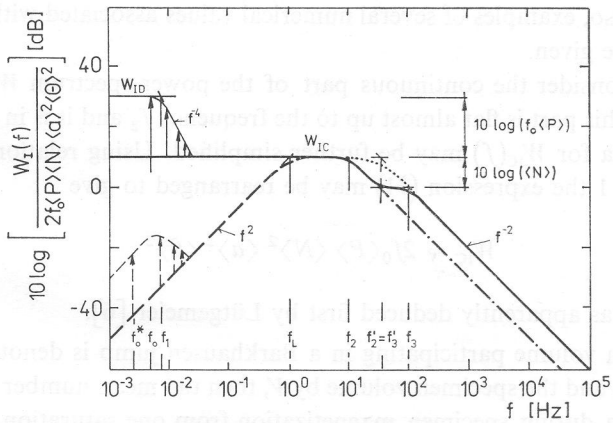

Fig. 5. Power spectrum of the Barkhausen voltage calculated for given hypothetical data. $f_0 = 1/T_0, f_1 = 1/(2\pi\langle\lambda\rangle), f_2 = 1/(2\pi\langle\varphi\rangle), f_3 = 1/(2\pi\langle\theta\rangle)$.

The expression (31) can be already displayed graphically. For this purpose we shall replace, using relation $\delta(\omega) = (2\pi)^{-1}\delta(f)$, ω with f, because f is more common in applications. Unfortunately, we did not find enough experimental data in the literature obtained only on one specimen and thus the following hypothetical values will be considered: $T_0 = 200$ s, $\langle P\rangle = 10^4$, $\langle N\rangle = 10$, $\langle\theta\rangle = 1{\cdot}6$ ms, $\langle\varphi\rangle = 16$ ms and

$\langle\lambda\rangle = 20$ s (in fact, the values of T_0, $\langle P\rangle$ and $\langle N\rangle$ are of the same order as those given in the work [5]). Using these data the expression (31) is schematically displayed in fig. 5 by the full line.

The values of parameters $\langle\theta\rangle$ and $\langle\varphi\rangle$ were selected intentionally in such a way as to demonstrate some characteristic features of the expression (31) and not to fit the measured spectrograms in every detail. Thus, for example, in experimentally found spectrograms the level of the continuous part decreases from the frequency f_2 uniformly. On the contrary, here the selection of parameters $\langle\theta\rangle$ and $\langle\varphi\rangle$ leads to a step in the spectrum between the frequencies f_2 and f_3. This was done to enable distinct separation of the contribution due to the single pulses and due to the pulse clustering. To fit the measured spectrograms better, it would be sufficient to choose, for example, either $\langle\theta'\rangle \geqq 5$ ms or $\langle\varphi'\rangle \leqq 5$ ms. This would give the form of the power spectrum plotted in fig. 5 by the chain or the dotted line respectively.

5. INTERPRETATION OF THE POWER SPECTRUM FORMULA

In the preceding section we arrived at the power spectrum formula based on statistical properties of Barkhausen pulses, that is, the Barkhausen voltage was treated on the "signal" level and we could say only very little about the underlying physical mechanism. It is the purpose of this section to give the derived results a certain physical meaning. Also, examples of several numerical values associated with the Barkhausen effect will be given.

Let us first consider the continuous part of the power spectrum $W_{IC}(f)$. As indicated in fig. 5, this part is flat almost up to the frequency f_2 and it is in this flat region that the formula for $W_{IC}(f)$ may be further simplified. Using relations (30) and assuming $\langle N\rangle \gg 1$ the expression (31) may be rearranged to give

$$W_{IC} = 2f_0\langle P\rangle \langle N\rangle^2 \langle a\rangle^2 \langle\theta\rangle^2 . \tag{32}$$

This formula was apparently deduced first by Lütgemeier [6].

If the domain volume participating in a Barkhausen jump is denoted by v (Barkhausen volume) and the specimen volume by V, then the mean number of Barkhausen jumps occurring during specimen magnetization from one saturation to the reversal one is given by

$$\langle P\rangle \langle N\rangle = \langle k\rangle V/\langle v\rangle , \tag{33}$$

where the factor $k < 1$ determines that part of the specimen volume magnetized by the irreversible jumps.

A Barkhausen jump will cause the change Δm in the magnetic moment of the specimen, the value of Δm being dependent not only on the Barkhausen volume v, but also on the angle between the direction of the coil axis and the vector of magnetization

J_s in the domain and on the angle between vectors of magnetizations on both sides of the domain wall. These angles are random variables in polycrystalline materials and very little can be said about their distributions at present. Therefore, the following approximate relation due to Stierstadt and Boeckh [10] will be used

$$\langle \Delta m \rangle = J_s \langle v \rangle . \tag{34}$$

The change in flux $\Delta\Phi$ through a pick-up coil following a change in magnetic moment Δm at the centre of the coil is given by [4]

$$\Delta\Phi = \int u(t)\,\mathrm{d}t = \frac{z\Delta m}{(r^2 + l^2)^{1/2}}, \tag{35}$$

where z is the total number of terns in the coil, r is the radius of the coil and $2l$ the length of the coil.

If the specimen is placed in the centre of a long solenoid ($l \gg r$) with z_1 terns per unit length, then (35) reduces to [1]

$$\int u(t)\,\mathrm{d}t = z_1 \Delta m . \tag{36}$$

In this work the pulse form was approximated by the exponential function (23). Hence

$$\int u(t)\,\mathrm{d}t = \int f(t, \boldsymbol{a})\,\mathrm{d}t = a\theta \tag{37}$$

and with respect to (34) and (36) we obtain

$$\langle a \rangle^2 \langle \theta \rangle^2 = z_1^2 \langle \Delta m \rangle^2 = z_1^2 J_s^2 \langle v \rangle^2 . \tag{38}$$

Thus, using (33) and (38) the expression (32) becomes

$$W_{IC} = 2 f_0 z_1^2 J_s^2 V \langle k \rangle \langle N \rangle \langle v \rangle . \tag{39}$$

Even if we do not have sufficient experimental data at our disposal, we believe that it will be useful to illustrate the derived expressions by numerical example. Just as in the preceding section, the following hypothetical data, partly taken over from [1] and [5], will be considered: $T_0 = 200$ s, $z_1 = 10^4\ \mathrm{m}^{-1}$, $J_s = 2$ T, $V = 6 \times 10^{-8}\ \mathrm{m}^3$, $\langle k \rangle = 0{\cdot}83$, $\langle N \rangle = 10$, $W_C = 10^{-12}\ \mathrm{V}^2\,\mathrm{s}$ and $\langle \theta \rangle = 1{\cdot}6$ ms. Then from equation (39) the average Barkhausen volume is $\langle v \rangle = 5 \times 10^{-13}\ \mathrm{m}^3$ and from equation (33) an average number of pulses that occur during specimen magnetization from one saturation to the reversal one is $\langle P \rangle \langle N \rangle = 10^5$, hence $\langle P \rangle = 10^4$. Finally, using equation (32) the mean pulse amplitude $\langle a \rangle = 6{\cdot}25\ \mu\mathrm{V}$ can be obtained.

Let us now briefly consider the discrete part $W_{ID}(f)$. Again, for frequencies $f < f_1$ the relations (30) can be used and thus the discrete components level will be given by

$$W_{ID} = 2 f_0^2 \langle P \rangle^2 \langle N \rangle^2 \langle a \rangle^2 \langle \theta \rangle^2 . \tag{40}$$

If (40) is compared with (32), it can be seen that the discrete part represents just as useful information on the Barkhausen effect as the continuous one. Unfortunately, in experiments it is usually either suppressed or at least omitted from analysis.

Using the values given above we obtain $W_{1D} = 50 \times 10^{-12}$ V^2 s. This level corresponds, for example, to a harmonic voltage of amplitude $A = 7$ μV. However, it is to be noted that in contrast to the continuous part level, which is proportional to f_0, the discrete components level is proportional to f_0^2. Therefore, if we suppose that the means $\langle P\rangle$, $\langle N\rangle$, $\langle a\rangle$ and $\langle\theta\rangle$ do not vary with magnetizing frequency f_0, then the amplitude of the harmonic voltage will be f_0'/f_0 times greater for magnetizing frequency f_0'. Thus, for example, while the harmonic voltage is "buried" in noise at $f_0 = 0{\cdot}005$ Hz ($\langle a\rangle = 6{\cdot}25$ μV, $A = 7$ μV), it is about four orders greater than the mean pulse amplitude at $f_0' = 50$ Hz ($\langle a\rangle = 6{\cdot}25$ μV, $A = 70$ mV).

6. DISCUSSION

The expression for the power spectrum of Barkhausen noise that was derived by Mazzetti and Montalenti [7] may be written for low magnetizing frequencies f_0 and using our symbols in the form

$$W_{1C}(\omega) = 2\langle v_1\rangle \langle|s(\omega, \boldsymbol{a})|\rangle^2 \left[1 + 2\frac{\langle N\rangle}{1 + \omega^2 \langle\tau\rangle^2}\right]. \tag{41}$$

Here $\langle v_1\rangle$ is an average number of pulses per second (the pulse density) and $\langle\tau\rangle$ is an average group length. Mazzetti and Montalenti have not considered periodicity in the pulse groups occurrence and therefore the term representing the line spectrum is not present in their formula.

In the paper [9] we introduced the 1st-order homogeneous group pulse process which is in a certain respect similar to the process considered by Mazzetti and Montalenti. Assuming that the number of pulses in groups was Poisson distributed, an expression for the power spectrum of this process was derived in the form (equation (39) in [9])

$$\mathcal{W}(\omega) = \langle v\rangle \langle N\rangle \langle|s(\omega, \boldsymbol{a})|^2\rangle + \langle v\rangle \langle N\rangle^2 \langle|s(\omega, \boldsymbol{a})|\rangle^2 |\chi_\varphi(\omega)|^2 + \\ + \langle v\rangle^2 \langle N\rangle^2 \langle|s(\omega, \boldsymbol{a})|\rangle^2 2\pi\delta(\omega)\,. \tag{42}$$

This formula can be further specified in the way introduced in section 4. Using (25) and (27) and considering the continuous part of the spectrum and $\omega \geqq 0$ equation (42) becomes

$$W_{1C}(\omega) = 2\langle v\rangle \langle N\rangle \langle|s(\omega, \boldsymbol{a})|\rangle^2 \left[1 + \frac{\langle N\rangle}{1 + \omega^2 \langle\varphi\rangle^2}\right]. \tag{43}$$

Here $\langle v\rangle$ is the mean group density, so that $\langle v\rangle \langle N\rangle = \langle v_1\rangle$. If (43) is compared with

(41), it can be seen that the two expressions differ in the value of the numerical factor in the square brackets and in the meaning of parameters $\langle\tau\rangle$ and $\langle\varphi\rangle$.

The expression for the continuous part of the power spectrum derived in this paper has the same form as (42) or (43). In this case $v_0\langle P\rangle\langle N\rangle = \langle v_1\rangle$.

Though the three mentioned processes are almost equivalent with respect to the continuous part of the power spectrum, only the 2nd-order periodic group pulse process incorporates the deterministic part and hence is suitable to approximate the Barkhausen voltage.

When forming the statistical model of the Barkhausen voltage in section 2 and when specifying the power spectrum formula in section 4, several assumptions were done, the most important of which concerned the polarity of the voltage pulses. Though the Barkhausen voltage consists of clusters of alternating polarity, only unipolar pulses were considered in this paper. This assumption influenced the form of both the discrete and the continuous parts of the power spectrum. Thus the basic component f_0 has double value $(f_0 = 2f_0^*)$ and the discrete spectrum is composed of components kf_0, where $k = 0, 1, 2, \ldots$. This, however, is not true for the Barkhausen voltage, in which only even harmonics are present $(k = 1, 3, 5, \ldots)$. The model also gives the continuous power spectrum, which is flat in the lowest frequency range from $f = 0$ to $f = f_2$. This, too, is not in agreement with experimentally found facts [2, 15], which indicate that the power spectrum grows as f^2 until a frequency $f_L < f_2$. An estimate of such a power spectrum form is plotted in fig. 5 by the dashed line.

Though a great number of works have been published on the Barkhausen effect recently, it still remains not fully explored and understood. Similarly, the model introduced here represents only a rough approximation and deserves further refinement. Nevertheless, it is hoped that it gives more exact insight into the composition of the Barkhausen voltage than the models used hitherto and thus makes it possible to put more precisely formulated questions for experimental works and to interpret the obtained results better.

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